

FINITE ELEMENT SIMULATION
OF
GROUNDWATER SYSTEM

by

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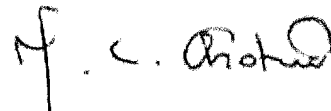
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(i)

CERTIFICATE

This is to certify that the thesis entitled, 'FINITE ELEMENT SIMULATION OF GROUNDWATER SYSTEM', being submitted by Mr. Raj Kishore Prasad to the Indian Institute of Technology, Delhi, India, for the award of the degree of DOCTOR OF PHILOSOPHY, is a record of bonafide research work carried out by him under our supervision and guidance. The thesis work, in our opinion, has reached the standard, fulfilling the requirements for DOCTOR of PHILOSOPHY degree. The research report and the results presented in this thesis have not been submitted, in part or in full, to any other University or Institute, for the award of any degree or diploma.



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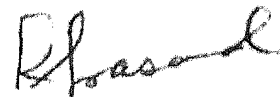
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(Raj Kishore Prasad)

ABSTRACT

Two versatile digital models, direct and inverse, have been developed to meet the long felt need for better aquifer analysis and management. The direct model is based on finite element approximation of the governing differential equation of groundwater flow. A general purpose computer program AQUFER has been developed for this purpose. It has a powerful solution algorithm, based on frontal house keeping technique. Crank-Nicholson type implicit time marching scheme has been incorporated. Eight noded, Isoparametric, quadrilateral elements have been used. The model has two distinct features. It automatically backs out the natural flux from known steady state water table/piezometric data by solving the Poisson's equation. This greatly reduces the uncertainty in input informations. The program has the capability to incorporate fully/partially penetrating stream with/without silt layer for studying stream-aquifer interaction appropriately. It has many other additional features that make it a versatile tool for better aquifer management. The capability of the program has been tested by applying it to a real life complex basin in Northern India for studying a variety of management issues.

To accelerate the calibrational phase of the model, sensitivity analysis of different groundwater flow parameters of the real life basin was carried out. The response of the system, as a consequence of systematic perturbations in the individual parameters, was studied on three statistical bases. The study, not only quickened the calibrational phase of the model building, but also provided an insight into relative sensitivities of various parameters.

The inverse model for the groundwater system identification problem is based on optimal control theory, coupled with finite element analysis. Variable metric algorithm due to Davidon-Fletcher and Powell has been employed for carrying out the unconstrained minimization. Use of an adjoint equation enables the gradients to be obtained in minimum computing time. A general purpose computer program OPTCON has been developed for this purpose. The program OPTCON was tested on a 65 variables identification problem and was found to work very satisfactorily. Convergence in the interior of the domain is achieved within a few iterations. Although, the boundary nodes having wider difference between true and guess values of the parameters, showed slightly slow convergence, the overall functioning of the scheme was found to be acceptable from practical point of view.

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