VARIATIONAL METHODS IN THE STABILITY ANALYSIS OF SLOPES

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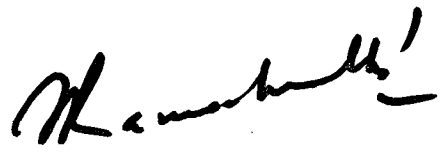
CERTIFICATE

This is to certify that the thesis entitled "Variational methods in the stability analysis of slopes" being submitted by Mr. C.G. Prabhakar Narayan to the Indian Institute of Technology, Delhi for the award of the degree of Doctor of Philosophy is a record of the bonafide research work carried out by him. Mr. C.G. Prabhakar Narayan has worked under my guidance for the submission of this thesis which to my knowledge has reached the requisite standard.

The thesis or any part thereof, has not been submitted to any other University or Institution for the award of any degree or diploma.



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SYNOPSIS

A mathematical technique is developed for analysing the stability analysis in terms of effective stresses. The stability equations are obtained based on limiting equilibrium conditions. The factor of safety defining the stable state of equilibrium is derived with respect to shear strength and follows Coulomb-Mohr failure criterion. The analysis considers the influence of effective inter-slice forces, and makes no a priori assumption regarding the shape of the slip surface, internal stress distribution or on the point of application of horizontal effective thrust line. It is emphasized that the normal stress distribution must be considered as a dependent variable, and must be obtained from the appropriate mathematical procedure. In the present thesis the stability analysis is transcribed as a minimization problem in the calculus of variations. The problem is to find the most critical slip surface which minimizes an appropriately defined factor of safety. A new function termed shear mobilizing factor is introduced which relates the effective inter-slice forces. Now the problem can be restated as to solve for the critical slip surface together with the so called shear mobilizing factor function which minimizes the factor of safety. Two methods namely, (a) indirect method (nonlocal variation); and (b) direct method (Raleigh-Ritz technique) are developed for obtaining the solutions of minimization problem. The numerical solutions are obtained

by examining the stability of specific problems. In few cases, results are compared with those obtained by existing analysis. Stability charts are presented which provide a convenient means to examine stability of slopes. Further analysis is carried out for slope sections with tension cracks. Numerical solutions are obtained using ICL 1909 series computer. The results are discussed and are mainly presented in the form of figures and tables.

From the numerical solutions obtained from calculus of variation, it is observed that variational technique can be more appropriately utilised for the rigorous stability analysis of slopes. The solutions obtained by indirect and direct methods in calculus of variations are in close agreement. The slip-surface obtained by variational technique resembles more closely to log-spiral, but, significantly deviates from slip-circle. The average factor of safety along the vertical inter-slice boundaries are generally greater than overall factor of safety. Normal stress distribution is observed to be dependent on the critical slip surface. The analysis satisfies all equilibrium, boundary, necessary and sufficient conditions. For the existence of a minimal, the minimal functional is verified for Legendre and Weierstrass conditions of positive definiteness and global extremal. Cases with tension cracks were also numerically analysed. Presence of tension crack slightly decreases the value of factor of safety. The line of action of

horizontal effective thrust line provides a means to locate the presence of tension crack. It is emphasized that for a more effective rigorous analysis the analysis should be carried out in terms of effective stresses. It is further stated that the variational techniques can be more conveniently utilised for analysing complex slope sections.

In this thesis, besides presenting the method of variational technique for the rigorous stability analysis of slopes, extensions and modifications to two of the recent methods in stability analysis of slopes have also been presented. This work include the modification of circular slip surface analysis presented by Bishop (1955) and Spencer (1967). In the former case a computational method is presented for quantifying inter-slice forces using active and passive earth pressure theory. In the latter case, is presented a modified computational algorithm for the stability analysis using slip-circle analysis presented by Spencer (1967). The computational algorithm developed is efficient and yields rapid solutions.

The thesis ends with a remark on the recommendations for future work using variational techniques.

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