LIE THEORY AND GENERATING FUNCTIONS

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CERTIFICATE

This is to certify that the thesis entitled "Lie Theory and Generating Functions" which is being submitted by Miss Sunita Jain for the award of the degree of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Delhi, is a record of bonafide research work that she has been doing for the last two and a half years under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other university or institute for the award of any degree or diploma.

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PREFACE

In view of the growing importance of generating functions, the piece of work in this thesis contains several generating functions based on different techniques.

It consists of an introductory chapter and four main chapters. A brief summary of the problem is presented at the beginning of each chapter.

The aim of the introductory chapter is to provide a survey of special functions used in the present work.

In chapter II an attempt is made to bring into use the ideas of Lie theory for obtaining generating functions for modified Laguerre polynomial $L_{m,n}^{(\gamma)}(x)$, hypergeometric functions ${}_2F_1[-m-n,\beta;\gamma;x]$, modified Jacobi polynomial $P_{m,n}^{(\alpha-n,\beta-n)}(x)$ and modified Laguerre polynomial $f_{m,n}^{\beta}(x) = (-1)^{m+n} L_{m,n}^{(-\beta-m-n)}(x)$.

In chapter III a new and powerful method based on the orthogonality of the exponentials for obtaining linear, bilinear and bilateral generating functions is introduced.

Then in chapter IV using Cauchy's Integral formula and series manipulation techniques certain results for Jacobi polynomials are obtained.

Finally, in chapter V theorems for generating functions have been given. These theorems being of a general nature unify generating functions of various polynomials and special functions

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