

# OPTIMIZATION OVER A POLYHEDRAL DOMAIN: SOME ASPECTS

BY  
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CERTIFICATE

This is to certify that the thesis entitled "OPTIMIZATION OVER A POLYHEDRAL DOMAIN: SOME ASPECTS" which is being submitted by Mrs. Vanita Verma for the award of the degree of DOCTOR OF PHILOSOPHY in MATHEMATICS to the Indian Institute of Technology, Delhi, is a bonafide record of research work done under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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*DEDICATED*

*TO*

*MY HUSBAND*

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## PREFACE

The present thesis is an outcome of the research work carried out by the author under the able supervision of Dr. M.C. Puri, Assistant Professor of Mathematics, Indian Institute of Technology, Delhi.

This thesis comprising of four chapters deals with integer programming, linear fractional programming and transportation problems.

Chapter I consisting of three sections is an introductory chapter. Origin and development of mathematical programming with special emphasis on integer programming, linear fractional programming and transportation problems is discussed in Section I.

Section II briefly surveys related work done by various authors in the relevant fields and finally, Section III gives a brief summary of the work carried out in subsequent three chapters of the thesis.

Chapter II consisting of two sections deals with integer programming problems. Algorithm for ranking integer feasible solutions in order of decreasing value of objective function is discussed in Section I.

Integer programming problems, where integer solutions are required to satisfy some fixed number of constraints out of a given set of additional constraints are dealt for linear as well as fractional objective function in Section II.

Discussion on effects of relaxing certain conventional assumptions in linear fractional programming problems forms the subject matter of Chapter III.

Section I of the chapter discusses algorithms due to Charnes-Cooper, Martos and Wolf with respect to identification of 'bad-points.

Discussion pertaining to identification of 'singular-points' in these algorithms is carried out in Section II.

Section III makes a comparative study of algorithms due to Charnes-Cooper and Wolf with respect to identification of optimal solution of linear fractional programming problem.

Chapter IV consists of three sections and discusses some special type of transportation problems.

Paradoxical situation for a linear fractional transportation problem has been analysed in Section I.

Section II deals with flow constrained transportation problem with lower and upper bounds on rim conditions.

Algorithm for solving bulk transportation problem, where for a given set of sources and destinations, that schedule is required, at which each source supplies material to at least one destination and each destination receives it from exactly one source, with given bulk transportation cost on each route, is developed in Section III of this chapter.

Numerical examples are given to illustrate various techniques discussed in the thesis.

References to papers/books mentioned in the thesis are given at the end and numbers in brackets [ ] refer to these references.

The numbering of various results is such that first number represents the chapter, second the section of that chapter and third the serial number of the result in that section.

Problems are numbered as P.a.b.c which refers to problem number c of section b of chapter a. Equations/ Inequations are represented by numbers such that first number represents chapter and second the serial number of equation/ inequation

The present thesis draws its subject matter from the following papers written by the author:

1. 'Bad-points' in Linear Fractional Program', published in 'Cahiers Du Centre D'Etudes De Recherche Operationnell Vol. 29, No. 1-2, 1987, Page No. 123-131.
2. 'On Martos' and Charnes-Cooper's Approach vis-a-vis 'Singular-points' published in 'Optimization', Vol. 20, No.4, 1989, Page No. 1-6.
3. 'Some Special Situations in Linear Fractional Programming Problem: An Algorithmic Comparison', published in 'Opsearch', Vol. 26, No.2, June 1989, page No. 96-107.

4. On Wolf's Method for Solving Linear Fractional Programming Problem', Communicated to 'Journal of Optimization Theory and Applications'.
5. 'Constrained Transportation Problem with Upper and Lower Bounds on Row-Availabilities and Destination-Requirements', published in 'Cahiers Du Centre D'Etudes De Recherche Operationnelle', Vol. 30, No.2-3,1988, page No. 177-190.
6. 'Constrained Integer Linear Programming', communicated to 'Journal of the Australian Mathematical Society Series B: Applied Mathematics'.
7. 'Constrained Integer Linear Fractional Programming Problem' accepted for publication in 'Optimization'.
8. 'Ranking in Linear Fractional Integer Programming Problem' accepted for publication in 'Zeitschrift für Operations Research'.
9. 'A Branch and Bound Algorithm for Cost Minimization Bulk Transportation Problem', Communicated to 'European Journal of Operations Research'.
10. 'On Paradox in Linear Fractional Transportation Problems' Communicated to 'Australian Society of Operations Research'.

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