SOME RANKING AND SELECTION PROCEDURES FOR MORE THAN ONE PARAMETER POPULATIONS

By
V. K. JAIN

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Department of Mathematics
INDIAN INSTITUTE OF TECHNOLOGY, DELHI
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DEDICATED TO

LORD MAHAVIRA

C E R T I F I C A T E

This is to certify that the thesis entitled "Some Ranking and Selection Procedures for more thanone Parameter Populations" which is being submitted by Mr. V.K. Jain for the award of degree of Doctor of Philosophy to the Indian Institute of Technology, Delhi is a record of bonafide research work carried out by him under my guidance and supervision.

The thesis has reached the standard fulfilling the requirement of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

(B.R. Handa)
Assistant Professor
Department of Mathematics
Indian Institute of Technology
Hauz Khas, New Delhi-110016.

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(V.K. JAIN)

A B S T R A C T

The majority of work on ranking and selection so far has been devoted to the selection of population with single largest (or smallest) parameter. Even if the populations involve more than one parameter the ranking has always been done on the basis of single parameter regarding other parameters as a nuisance parameters. These other parameters present in the population may cause considerable difficulties in the development of selection procedures. Further, even if other parameters were required to contribute the ranking has been usually done in terms of a single function of the parameters associated with each population. In practice one encounters situations where every parameter is important on its own right and one has to formulate goals which select population that is in some sense best with respect to all the parameters. The selection procedures for such goals have been rarely attempted in the past. Further, in the method of paired comparison the best item could be defined in terms of preference probabilities treated as parameters. This is basically a multiparameter selection problem. The emphasis here is in the development of a economical paired comparison design for selecting the best population. There is a plenty of scope of work in this direction.

The aim of this thesis is to develop new selection procedures for populations which involve more than one parameter. Specifically, we have dealt with the selection of two parameter normal distribution, two parameter exponential distribution and multiparameter situation for ranking of treatment, items etc. in terms of preference probabilities in the method by paired comparison. Some new goals considered treat the two parameters of exponential and normal population as independent parameters and rank the populations with respect to both the parameters simultaneously. The selection procedures for these goals developed in the thesis are generally two stage procedures and combine subset and indifference zone approaches, A few new single stage subset selection procedures for selection of best location parameter in presence of unknown and unequal scale parameter have been also developed. Finally, two stage selection procedures for a selection of best population, that use data partly from knock out design and partly from round robbin design have been proposed. Because more general parameter spaces or indifference zones are needed to handle goals involving two or more independent parameters the expressions of probability of correct selection in these spaces are generally cumbersome and finding exact infimum of probability of correct selection is usually difficult. Consequently we have often based our selection procedures on an appropriate lower bound of probability of correct selection.

Specifically Chapter I is an introductory chapter.

In Chapter II the following two goals have been considered

(i) Select the population as best which has largest mean amongst the means of those populations whose variances are less than or equal to the variances of at least r out of k given populations, where r is a specified integer.

(ii) Select the population as best which has minimum variance amongst the variances of those populations whose means are larger than or equal to the means of at least r out of k given populations.

In Chapter III we have developed the selection procedures for the selection of normal and exponential populations on the basis of single parameter treating other parameter as unknown and unequal in almost all cases. The two stage selection procedure has been given under indifference zone formulation while a few single stage procedures have been given under subset selection approach.

In Chapter IV the selection procedures have been developed for two parameter exponential populations for goals similar to those in Chapter II for normal population.

Finally in Chapter V two selection procedures for the goal of selection of best population are considered when the data is through a combined knockout and round robbin design. The selection procedures developed are two-stage procedures. We have compared our two selection procedures on the basis of expected number of paired comparisons needed to complete the selection.

C O N T E N T S