

NESTED STRUCTURES FOR FIR FILTERS

**BY
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CERTIFICATE

This is to certify that the thesis entitled 'Nested Structures for FIR Filters' being submitted by Anil Mahanta to the Electrical Engineering Department, Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy, is a record of bonafide research work carried out by him under my supervision and guidance and in my opinion, it has reached the standard fulfilling the requirements of the regulation relating to the degree.

The results contained in this thesis have not been submitted to any other institute for the award of any degree or diploma.

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ABSTRACT

For efficient and economic realization of digital filters, structures having low coefficient sensitivity and roundoff noise are desirable. This thesis presents a class of structures for FIR filters, which exhibits reduced coefficient sensitivity and superior roundoff noise characteristics as compared to the commonly used direct form realization. These structures, designated as 'nested structures', are derived from the transfer function polynomial by decomposing it into a sum of simpler polynomials, and then expressing each component polynomial in the nested form. Due to nesting and subsequent scaling, the dynamic range is reduced considerably with the result that using fixed-point arithmetic, these structures in fact realize the accuracy of the floating-point implementation. The other major advantage that results from the nested configuration is that most of the noise appears at the output in a heavily attenuated form, and thus these structures should be particularly attractive for higher order filters. It is shown that the manner in which the transfer function is decomposed and the order in which the coefficients are nested, have significant bearing on the output noise. The optimum structure which achieves the minimum roundoff noise can be found ; however, the search in this direction is not

as involved as in the cascade form ; in most cases, near optimum results are easily obtained by inspection of the impulse response coefficients.

The class of structures reported in this thesis may be considered to be a hybrid between the fully-nested structure and the usual direct form realization. Thus these structures offer considerable flexibility in the choice of realization for a given specification vis-a-vis the accuracy and speed requirements and the overall hardware complexity.

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