

ON OPTIMAL CONTROL OF STOCHASTIC  
DYNAMICAL SYSTEMS

by

BANDARU VENKATA RAJARAO  
B.E.(Hons), M.Tech.

Thesis submitted in partial fulfilment of  
the requirements for the degree of  
Doctor of Philosophy

to the  
Indian Institute of Technology, Delhi

Department of Electrical Engineering  
Indian Institute of Technology  
New Delhi-29 INDIA

December 1970

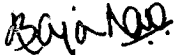
## PREFACE

The thesis is based on results of the investigations carried out by the author at the Indian Institute of Technology, Delhi, over the period December 1967 to November 1970. All of these studies have been concerned with problems of optimal control of stochastic dynamical systems. Except the investigations reported in chapter 4, all the other studies are concerned with nonlinear systems. Attempts have been made to develop suboptimal solutions for the nonlinear control problem. In chapter 4, however, an optimal solution is presented for linear stochastic systems involving process time delay.

I would like to express my gratitude to my thesis supervisor Professor A. K. Mahalanabis, whose work and encouragement provided the motivation and support for the thesis. I would also like to thank my colleagues Mr. S. Purkayastha and Mr. H.S. Kang for their suggestions and useful discussions. Thanks are also due to the Head of Electrical Engineering Department for providing the facilities in the Department.

Finally I wish to thank my wife Lakshmi for her patient understanding, help and encouragement throughout the course of this endeavour.

New Delhi-29  
December 12, 1970.

  
B.V. RAJA RAO

## C O N T E N T S

CHAPTER I	INTRODUCTION	1
1.1	Some results in Deterministic Optimal Control Theory.	2
1.2	Optimal Stochastic Control	7
1.3	Statement of the Problems	19
1.4	Organisation of the Thesis	24
CHAPTER II	MINIMUM VARIANCE CONTROL OF DISCRETE TIME NONLINEAR STOCHASTIC SYSTEMS	29
2.1	General Discussion	29
2.2	Problem Formulation	30
2.3	Method of Solution	32
2.4	Suboptimal Solutions	36
2.5	Examples	44
2.6	Conclusions	47
CHAPTER III	MINIMUM VARIANCE CONTROL OF NONLINEAR CONTINUOUS TIME STOCHASTIC SYSTEMS	54
3.1	General Discussion	54
3.2	Problem Formulation	56
3.3	Method of Solution	58
3.4	Results based on Taylor's Series	62
3.5	An Alternative Approach	67
3.6	Examples	70
3.7	Conclusions	73

CHAPTER IV	MINIMUM VARIANCE CONTROL OF LINEAR DISCRETE TIME STOCHASTIC SYSTEMS WITH TIME DELAY	77
4.1	General Discussion	77
4.2	Problem Formulation	78
4.3	Method of solution	81
4.4	An Alternative Approach	84
4.5	An Example	88
4.6	Conclusions	89
	Appendix	90
CHAPTER V	ON OPTIMIZATION OF NONLINEAR STOCHASTIC SYSTEMS SUBJECT TO QUADRATIC CRITERION	97
5.1	General Discussion	97
5.2	Problem Formulation	98
5.3	Linear systems	99
5.4	Statistical Linearization	104
5.5	Stochastic Quadratic Approximation	107
5.6	Examples	110
5.7	Conclusions	111
CHAPTER VI	ON OPTIMIZATION OF LINEAR STOCHASTIC SYSTEMS WITH NONLINEAR NOISY MEASUREMENTS SUBJECT TO QUADRATIC CRITERION	117
6.1	General Discussion	117
6.2	Problem Formulation	118
6.3	Method of Solution	119
6.4	Solution of the Filtering Problem	121
6.5	Solution of the suboptimal Control Problem	123
6.6	An Example	125
6.7	Conclusions	127

CHAPTER VII CONCLUSIONS AND SUGGESTIONS	130
7.1 Conclusions	130
7.2 Suggestions for further work	132
REFERENCES	134