Numerical investigation of control strategies in turbulent Taylor-Couette Flows

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Certificate

This is to certify that the thesis entitled **"Numerical investigation of control strategies in turbulent Taylor-Couette Flows"**, being submitted by **Obaidullah Khawar** to the Indian Institute of Technology Delhi for the award of the degree of **Doctor of Philosophy**, is a record of the bonafide research carried out by him which has been prepared under our supervisions in conformity with the rules and regulations of the Indian Institute of Technology Delhi. The research reports and results presented in the thesis has not been submitted for any degree or diploma in any other university or institute.

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Abstract

The flow between the annular region of concentric rotating cylinders is referred to as Taylor-Couette flow. Applications of such annular flows are ubiquitous in high-speed electrical and turbo-machines, process industries, and astro and geophysical systems. An understanding of Taylor-Couette flow is important both from an industrial application point of view and also from the viewpoint of fundamental fluid dynamics. In high-speed electrical and turbo-machines, skin-friction drag a very critical parameter. The overall aim of this thesis is to explore the role of active control strategies in achieving sustained control of turbulence. Various control strategies, namely radial temperature gradient, opposition control, and axial oscillation of the inner cylinder have been investigated.

First simple rotating Taylor-Couette flows with radial temperature gradient are studied. In this study gravity and rotation axis are mutually orthogonal. An In-house finite-difference-based incompressible solver is used to perform three dimensional numerical simulations in cylindrical coordinates in a fixed frame of reference. The effect of increase in rotation rate ($1000 \leq \mathrm{Re} \leq 5000$) in tandem with thermal stratification ($0 \leq \mathrm{Ri} \leq 0.3$) is investigated. For neutrally buoyant cases, flow statistics and dynamics reveal that on increasing the rotation rate more intense turbulence is observed, as depicted in azimuthal velocity fluctuations and Reynolds shear stresses. Near-wall streaks form a herringbone-like pattern, and the streak spacing is uniform in the axial direction. On heating the outer wall of the cylinder, since buoyancy now acts in both radial and azimuthal directions, a complicated flow phenomenon is observed. For weak to moderate buoyancy, due to the interaction of inertial and thermal buoyancy forces, the near-wall streak spacing

decreases. The streaks are observed throughout the axial domain. The spatial density of the vortical structure increases. Heating of the outer cylinder results in more intense streaks and coherent structures in the half-circumferential domain due to unstable stratification aiding turbulence, while in the other half-domain, stable stratification mitigates turbulence.

In counter-rotating Taylor-Couette flows, the effect of thermal stratification ($0 \leq \mathrm{Ri} \leq 0.4$) in tandem with an increase in rotational speed of the inner and the outer cylinder is investigated such that $Re_i = Re_o$ (ranges from 1000 to 5000). Large Taylor rolls are observed with scales much smaller than the gap width, unlike SRTC flows where Taylor rolls occupy full gap width. Vortical structures are observed near the inner and outer wall, while the core is almost vortex free. On heating, depending on the interaction of inertial and thermal buoyancy force, thermal stratification leads to suppression and enhancement of turbulence in respective halves, as observed in simple rotating Taylor-Couette flows.

In opposition control, various velocity control strategies are numerically investigated. The idea is to reduce skin friction by introducing blowing and suction near the walls. Wall-normal velocity control shows maximum skin-friction drag reduction at $r^+ = 15$. The spatial skipping of points in azimuthal and axial directions, as well as temporal skipping, is performed for wall-normal velocity control in order to check its experimental realizability. A virtual wall is formed between the real wall and the detection plane inhibiting vertical transport of momentum for all the cases exhibiting skin-friction drag reduction. A marked reduction in the spatial density of vortical structures is observed.

In axial oscillation of the inner cylinder, numerical simulation is performed for wide gap T-C flows. The maximum skin-friction drag reduction occurs at an optimal oscillating period. The skin-friction drag reduction is attributed to the formation of Stoke's layer, which affects the near-wall self-sustaining cycle. Further, the effect of oscillating amplitude is investigated for a fixed optimal oscillating period. A marked reduction in turbulent intensities, Reynolds stress and vortical structures is observed as amplitude increases. Beyond a threshold amplitude, transition to a new regime occurs leading to enhanced turbulence and an increase in skin-friction drag.

सार

संकेंद्रित घूर्णन सिलेंडरों के कुंडलाकार क्षेत्र के बीच के प्रवाह को टेलर–कूएट प्रवाह कहा जाता है। इस तरह के कुंडलाकार प्रवाह के अनुप्रयोग उच्च गति में सर्वव्यापी हैं जैसे विद्युत और टर्बो–मशीन, प्रक्रिया उद्योग, और खगोल और भूभौतिकीय प्रणालियाँ। टेलर–कूएट प्रवाह की समझ औद्योगिक अनुप्रयोग एवं मौलिक द्रव गतिकी दोनों दृष्टिकोण से महत्वपूर्ण है। उच्च गति में विद्युत और टर्बो –मशीन, त्वचा–घर्षण एक बहुत ही महत्वपूर्ण पैरामीटर है। कुल मिला कर इस थीसिस का उद्देश्य निरंतर प्राप्त करने में सक्रिय नियंत्रण रणनीतियों की भूमिका का पता लगाना है।

पहले सरल घूर्णन टेलर–कूएट प्रवाह के साथ रेडियल तापमान प्रवणता का अध्ययन किया गया है। इस अध्ययन में गुरुत्वाकर्षण और घूर्णन अक्ष परस्पर ओर्थोगोनल हैं। इन–हाउस परिमित अंतर–आधारित असंपीड़ित सॉल्वर का उपयोग तीन आयामी संख्यात्मक प्रदर्शन करने के लिए एक संदर्भ के एक निश्चित फ्रेम में बेलनाकार निर्देशांक में सिमुलेशन किया गया है। वृद्धि का प्रभाव थर्मल स्तरीकरण ($0 \le Ri \le 0.3$) के साथ मिलकर रोटेशन दर ($1000 \le Ri \le 5000$) में जांच की गई है। तटस्थ रूप से उत्प्लावक मामलों के लिए, प्रवाह सांख्यिकी और गतिकी से पता चलता है कि घूर्णन दर में वृद्धि से अधिक तीव्र अशांति देखी जाती है, जैसा कि अज़ीमुथल वेग के उतार–चढ़ाव में दर्शाया गया है और रेनॉल्ड्स कतरनी तनाव में। निकट–दीवार की धारियाँ एक हेरिंगबोनिक पैटर्न बनाती हैं, और स्ट्रीक रिक्ति अक्षीय दिशा में एक समान होती है। गरम करने पर बेलन की बाहरी दीवार, चूंकि उत्प्लावकता अब रेडियल और अज़ीमुथल दोनों दिशाओं में कार्य करती है, एक जटिल प्रवाह घटना देखी जाती है। कमजोर से मध्यम उछाल के लिए, देय जड़त्वीय और ऊष्मीय उत्प्लावकता बलों की परस्पर क्रिया के लिए, निकट–दीवार स्ट्रीक रिक्ति कम हो जाती है। धारियाँ पूरे अक्षीय क्षेत्र में देखी जाती हैं। वर्टिकल संरचना के स्थानिक घनत्व में वृद्धि होती है। बाहरी सिलिंडर को गर्म करने से अधिक तीव्र हो जाती है अर्ध –परिधि क्षेत्र में धारियाँ और सुसंगत संरचनाएँ जिसका मुख्य अस्थिर स्तरीकरण जो कि अशांति पैदा करता है।

काउंटर–रोटेटिंग टेलर–कूएट प्रवाह में, थर्मल स्तरीकरण का प्रभाव ($0 \le Ri \le 0.4$) आंतरिक और बाहरी सिलेंडर की घूर्णी गति में वृद्धि के साथ मिलकर इस तरह से जांच की जाती है कि $Re_i = Re_o$ (1000 से 5000 तक की सीमा)। बड़े टेलर रोल जो कि SRTC प्रवाह के विपरीत जहां टेलर रोल पूर्ण अंतराल चौड़ाई पर कब्जा कर लेते हैं। आंतरिक और बाहरी दीवार के पास ऊर्ध्वाधर संरचनाएं देखी जाती हैं, जबकि कोर लगभग भंवर मुक्त है। गर्म करने पर, जड़त्वीय की परस्पर क्रिया पर निर्भर करता है और थर्मल उछाल बल, थर्मल स्तरीकरण दमन और वृद्धि की ओर जाता है संबंधित हिस्सों में अशांति, जैसा कि साधारण घूर्णन टेलर–कूएट प्रवाह में देखा गया है।

विपक्षी नियंत्रण में, विभिन्न वेग नियंत्रण रणनीतियों की संख्यात्मक रूप से जांच की गई है। विचार दीवारों के पास ब्लोइंग और सक्शन शुरू करके त्वचा के घर्षण को कम करना है। दीवार–सामान्य वेग नियंत्रण $r+ = 15$ पर अधिकतम त्वचा–घर्षण ड्रैग कमी को दर्शाता है। अज़ीमुथल और अक्षीय दिशाओं में बिंदुओं का स्थानिक लंघन, साथ ही अस्थायी लंघन, इसकी प्रयोगात्मक साध्यता जांच के लिए दीवार–सामान्य वेग नियंत्रण के लिए किया गया

है। वास्तविक दीवार और डिटेक्शन प्लेन के बीच एक आभासी दीवार बनती है जो कि त्वचा–घर्षण ड्रैग को प्रदर्शित करने वाले सभी मामलों के लिए गति के लंबवत परिवहन को रोकती है। भंवर संरचनाओं के स्थानिक घनत्व में उल्लेखनीय कमी देखी गई है।

आंतरिक सिलेंडर के अक्षीय दोलन में, विस्तृत के लिए संख्यात्मक अनुकरण अंतर टी–सी प्रवाह के लिए किया गया है। अधिकतम त्वचा–घर्षण ड्रैग कमी एक इष्टतम दोलन अवधि में होती है। स्टोक के गठन के लिए त्वचा–घर्षण ड्रैग कमी को जिम्मेदार ठहराया गया है परत, जो निकट–दीवार के आत्मनिर्भर चक्र को प्रभावित करती है। इसके अलावा, दोलन का प्रभाव एक निश्चित इष्टतम दोलन अवधि के लिए आयाम की जांच के लिए किया गया है। अशांत तीव्रता, रेनॉल्ड्स तनाव और भंवर संरचनाओं में उल्लेखनीय कमी को आयाम के बढ़ने से देखा जाता है। थ्रेशोल्ड आयाम से परे, एक नए शासन में संक्रमण होता है जिसके कारण बढ़ी हुई अशांति और त्वचा–घर्षण ड्रैग में वृद्धि होती है।

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Nomenclature

Notations

Δt $\equiv$ time step

i, j, k $\equiv$ local grid indices

p^* $\equiv$ corrected pressure

p^n $\equiv$ pressure at previous time level

p^{n+1} $\equiv$ pressure at new time level

t $\equiv$ time scale

$\mathbf{U}$ $\equiv$ velocity vector

P $\equiv$ pressure

Θ $\equiv$ temperature

$\tilde{\mathbf{U}}$ $\equiv$ predicted velocity vector

$\tilde{\Theta}$ $\equiv$ predicted temperature

$\mathbf{U}^{'}$ $\equiv$ corrected velocity vector

$\mathbf{U}^n$ $\equiv$ velocity vector at previous time level

$\mathbf{U}^{n+1}$ $\equiv$ velocity vector at new time level

U_i or u_Θ $\equiv$ velocity scale

r, θ, z $\equiv$ direction in cylindrical coordinates

u_r, u_θ, u_z $\equiv$ velocity in r, θ and z directions

$u_r{}^{'}$, $u_\theta{}^{'}$, $u_z{}^{'}$ $\equiv$ fluctuating velocities in r, θ and z directions

1D $\equiv$ one dimension

2D $\equiv$ two dimension

3D	$\equiv$	three dimension
D	$\equiv$	gap width
r_i, r_o	$\equiv$	radii of the inner and outer cylinder
T_i, T_o	$\equiv$	inner and outer cylinder temperature
L_z	$\equiv$	axial length of the domain
g	$\equiv$	gravitational acceleration
Re	$\equiv$	Reynolds number
Re_τ	$\equiv$	frictional Reynolds number
Re_i	$\equiv$	Reynolds number of the inner cylinder
Re_o	$\equiv$	Reynolds number of the outer cylinder
Re_s	$\equiv$	Reynolds number based on shear
Ri	$\equiv$	Richardson number
Pe	$\equiv$	cell-Peclet number
$u_{\Theta s}$	$\equiv$	shear velocity
$\tilde{S}$	$\equiv$	typical shear
$\tilde{r}$	$\equiv$	typical radius
Ta	$\equiv$	Taylor number
ΔT	$\equiv$	temperature difference between the inner and outer cylinder
Pr	$\equiv$	Prandtl number
T	$\equiv$	oscillating period
A	$\equiv$	oscillating amplitude
δ_s	$\equiv$	Stoke's layer thickness
h	$\equiv$	half-channel height

Greek Letters

η	$\equiv$	radius ratio, $\left(\frac{r_o}{r_i}\right)$
Γ	$\equiv$	aspect ratio
ω_i	$\equiv$	angular speed of rotation of inner cylinder
ω_o	$\equiv$	angular speed of rotation of outer cylinder

ϵ^n ≡ iteration-error at current time-level

ξ ≡ computational space

ρ ≡ density of the fluid

ν ≡ kinematic-viscosity

τ ≡ wall-shear stress

β ≡ coefficient of thermal volume expansion

Other Symbols

∇ ≡ gradient operator

∇^2 ≡ laplace operator

∂ ≡ partial derivative

∂^2 ≡ partial double derivative

Abbreviations

DNS ≡ Direct Numerical Simulation

LES ≡ Large Eddy Simulation

RANS ≡ Reynolds-averaged Navier-Stokes

FDM ≡ Finite Difference Methods

MPI ≡ Message Passing Interface

API ≡ Application programming interface

PPE ≡ pressure Poisson equation

PCPE ≡ pressure correction Poisson equation

SMAC ≡ Simplified Marker and Cell

PIV ≡ Particle Image Velocimetry

LDV ≡ Laser Doppler Velocimetry

SRTC ≡ Simple rotating Taylor-Couette

CRTC ≡ Counter rotating Taylor-Couette

RB ≡ Rayleigh-Bénard

RAM ≡ Random access memory

CPU ≡ central processing unit

SMP ≡ shared memory parallel

RMS ≡ root mean square

TKE ≡ turbulent kinetic energy

PSD ≡ power spectral density

T-C ≡ Taylor-Couette

SFDR ≡ Skin-friction drag reduction