FLOWS AND WAVES IN CONTINUUM MECHANICS

by

SARBESWAR ROUT

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IN THE EVER LOVING
MEMORY OF MY
MOTHER

CERTIFICATE

This is to certify that the thesis entitled:

"FLOWS AND WAVES IN CONTINUUM MECHANICS"

which is being submitted by Mr. Sarbeswar Rout, Research Scholar, Mathematics Department to the Indian Institute of Technology, Delhi, for the award of the DEGREE OF DOCTOR OF PHILOSOPHY in MATHEMATICS, is a record of bonafide research work carried out by him under our guidance and supervision and has fulfilled all the requirements for the submission of this thesis.

The results contained in this thesis have not been submitted in part or full, to any other University or Institute for the award of any degree or diploma.

O.P. BHUTANI
Professor
Department of Mathematics
I.I.T., New Delhi

PREM KUMAR
Professor
Department of Mathematic
I.I.T., New Delhi

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SARBESWAR ROUT

ABSTRACT

The thesis entitled "Flows and Waves in Continuum Mechanics" comprises seven chapters in all and is divided into two parts. Chapter 1 is of an introductory nature and Chapters 2-5 constitute Part I of the thesis and deal with mass transfer in laminar boundary layers with/without heat transfer. The second part that contains only two chapters deals with the problems of wave propagation in a rotating elastic medium with electromagnetic interaction and/or no thermal relaxation effect. The detailed chapterwise plan of the thesis is as follows:

Chapter 1, outlines the basic concepts of boundary layer flows of single component and two-component fluid in the context of their importance in the problems of heat and mass transfer. This is followed by a section on the critical survey and development of the numerical methods used for solving the boundary layer equations, with special reference to the problems dealt in Chapters 2-5. Further, for the problems of wave propagation dealt in Chapters 6-7, a brief resume of the various techniques utilised, to

date, is given and their limitations are pointed out. The chronological survey of the related works on wave propagation gives a clear picture of the problems dealt in the thesis.

In Chapter 2, an attempt is made to study the natural convection boundary layer flow over a vertical, isothermal flat plate subjected to the injection of foreign gases of various molecular weights with diffusion-thermo (DT). For this purpose, the governing differential equations are transformed to a highly complex system of nonlinear ordinary differential equations. Besides, one of the boundary conditions turns out to be a mixed one. In order to solve the resulting two-point boundary value problem we have carried over the technique of quasilinearization. This has resulted in a number of interesting results concerning the mass transfer, the direction of the flow and the heat transfer. More specifically, it is observed that, under certain conditions the injection of light gases like Hydrogen and Helium are more effective in transferring the heat. Further, as the blowing velocity increases for these gases, the direction of heat transfer is reversed. Moreover, for the gases like Neon and Argon, the constant property heat transfer results provide good approximation to the variable property heat transfer results, when the blowing rate is small. For higher blowing rates, the flow reversal is noticed for the heavier gas, Argon. It turns out that DT does not influence the mass transfer significantly.

In Chapter 3, we have studied the effect of vectored surface mass transfer on the flow of an incompressible laminar boundary layer over an axisymmetric cone. Due to the boundary condition imposed on the flow, the system of governing equations is transformed into a non-similar one. Consequently, the Box method as developed by Keller and Cebeci is used to solve the resulting system of partial differential equations. The technique brings out clearly the effect of the tangential component of surface mass transfer velocity and the cone angle.

In Chapter 4, the investigation is carried out on the effects of vectored surface mass transfer on the heat transfer in the laminar natural convection boundary layers over isothermal bodies of arbitrary contours. The governing equations are first nondimensionalised and then transformed. Unlike the case in Chapter 3, where the boundary condition yielded the non-similar system of equations, herein, the body shape is responsible for the non-similar form of the transformed equations. Once again, the Keller Box technique is utilised to solve the resulting system of governing equations. The results are tabulated for different body orientations, aspect ratios and various values of tangential and normal components of the mass transfer parameter. It is observed that the slender body transfers more heat and the tangential component of the mass transfer velocity influences the heat transfer significantly. Further, wherever possible comparisons have been made with the available theoretical and experimental results.

In Chapter 5, the scope of the problem dealt in the previous chapter is enlarged to study the effect of uniform heat flux at the surface. The governing equations are transformed after non-dimensionalising them by using a different set of non-dimensional variables. As expected the resulting equations turn out to be non-similar and are due to the body shape and the boundary conditions. The results are presented for different body shapes, aspect ratios and various values of the tangential and normal components of the mass transfer. It is seen that the normal suction increases the heat transfer parameter, while the normal injection decreases it. The tangential component of the down-stream surface mass transfer velocity increases the heat transfer parameter, while the opposite is true for the tangential component of the upstream surface mass transfer velocity. For the case of zero mass transfer, the results are compared with the available results.

Using the linearized system of equations, we have examined in Chapter 6, the problem of propagation of small disturbances and weak waves in a thermo-elastic rotating medium with thermal relaxation and subjected to an external magnetic field. The study provides many a remarkable conclusion concerning the waves of different order appearing in the problem, their inter-actions, dampings, relative times of dominance and diffusion. Further, using the characteristic coordinate system, the problem of weak waves is analysed. The last case does not seem to have received any attention, whatsoever, in the context of non-conventional continuum mechanics.

Keeping in view the limitations of the plane wave solution dealt in Chapter 6, we have in Chapter 7, investigated via Lighthill technique, the waves due to a localized periodic source of disturbance in an unbounded isotropic, conducting medium permeated by an external magnetic field and rotating with a uniform angular velocity. The effect of the applied magnetic field and rotation is to introduce the coupling of various wave modes, which in turn, render the medium anisotropic. In all, there are three distinct modes of waves and the technique utilised gives a complete geometrical description of these waves.

CONTENTS

		PAGE NO.
ABSTRACT		i-vi
CHAPTER-1:	CRITICAL SURVEY AND THE SCOPE OF THE WORK	
1.1	Introduction	1
1.2	Boundary Layer Theory	2
1.3	Methods of Solution	7
1.4	Magneto-Elastic and Magneto-Thermo-Elastic Waves	30
1.5	Fundamentals of Magneto-Elasticity and Magneto-Thermo-Elasticity	30
CHAPTER-2:	DIFFUSION-THERMO EFFECT IN LAMINAR NATURAL CONVECTION BOUNDARY LAYER OVER A VERTICAL FLAT PLATE WITH THE INJECTION OF FOREIGN GASES OF VARIOUS MOLECULAR WEIGHTS	
2.1	Introduction	40
2.2	Equations of Motion	42
2.3	Method of Solution	51
2.4	Numerical Results and Discussion	59

PAGE NO.

CHAPTER-3: LAMINAR INCOMPRESSIBLE BOUNDARY LAYER FLOW OVER AN AXI-SYMMETRIC CONE WITH VECTORED SURFACE MASS TRANSFER

3.1 Introduction 75

3.2 Equations of Motion 77

3.3 Method of Solution 81

3.4 Numerical Results and Discussion 87

CHAPTER-4: LAMINAR NATURAL CONVECTION BOUNDARY LAYERS OVER TWO-DIMENSIONAL AND AXI-SYMMETRIC ISOTHERMAL BODIES OF ARBITRARY SHAPE WITH VECTORED SURFACE MASS TRANSFER

4.1 Introduction 92

4.2 Equations of Motion 94

4.3 Numerical Results and Discussion 101

4.3A Flow Around Two-Dimensional Bodies 102

4.3B Flow Around Axi-Symmetric Bodies 109

CHAPTER-5: LAMINAR NATURAL CONVECTION BOUNDARY LAYERS OVER TWO-DIMENSIONAL AND AXI-SYMMETRIC BODIES OF ARBITRARY SHAPE WITH VECTORED SURFACE MASS TRANSFER AND UNIFORM SURFACE HEAT FLUX

5.1 Introduction 121

5.2 Equations of Motion 122

5.3 Numerical Results and Discussion 127

PAGE NO.

5.3A Flow Around Two-Dimensional Bodies 127

5.3B Flow Around Axi-symmetric Bodies 130

CHAPTER-6: ON THE PROPAGATION OF SMALL DISTURBANCES AND WEAK WAVES IN MAGNETO-THERMO-ELASTIC ROTATING MEDIA WITH THERMAL RELAXATION

6.1 Introduction 137

6.2 Equations of Motion 138

6.3 General Character of Various Modes 147

6.4 Weak Wave Analysis 155

CHAPTER-7: ON THE PROPAGATION OF WAVES DUE TO A LOCALIZED PERIODIC SOURCE IN AN UNBOUNDED ISOTROPIC ROTATING MEDIUMPERMEATED BY A MAGNETIC FIELD

7.1 Introduction 161

7.2 Equations of Motion 163

7.3 Asymptotic Solution 170

7.4 Wave Characteristics Via Wave Number Surface 173

BIBLIOGRAPHY 179