EIGENSUBSPACE BASED ALGORITHMS FOR POLYSPECTRAL AND MULTIDIMENSIONAL SIGNAL PARAMETER ESTIMATION OF HARMONIC PROCESSES

By

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To my parents

CERTIFICATE

This is to certify that the thesis entitled "**Eigensubspace Based Algorithms for Polyspectral and Multidimensional Signal Parameter Estimation of Harmonic Processes**", being submitted by Harish Parthasarathy to the Department of Electrical Engineering, Indian Institute of Technology, New Delhi, for the award of the degree of Doctor of Philosophy, is a bonafide research work carried out by him under our supervision. The results contained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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Abstract

Generally, higher order statistics and spectra, in conjunction with the second order statistics and the power spectrum, provide additional information regarding statistical properties of random processes and parameters of systems, to what can be obtained using only the latter. As such, they enable us to obtain more accurate characterizations of signals and systems. An important motivation behind the use of polyspectra is based on the fact that the power spectrum is phase blind. It gives only the distribution of power among the harmonic components of a process, but suppresses *all* information regarding phase relations and other statistical dependencies that might exist among these components. On the other hand, any polyspectrum of order greater than two is phase sensitive, and hence carries some phase information about the process. As such, polyspectra have been used by researchers to quantify phase relations. Since phase relations among the harmonic components of a process are typically due to nonlinear interactions, polyspectra being phase sensitive, can also be used to characterize nonlinearities via phase relations.

Of particular interest is the study of the polyspectra of harmonic processes. This is because, harmonic processes serve as prototypes for many commonly observed physical processes, especially when the number of dominant frequencies in the signal is small. Such signals appear in a wide variety of applications like harmonic retrieval, bearing estimation of narrowband sources, echo resolution, time delay estimation and transient response analysis. Much attention has been devoted in the literature to frequency estimation of harmonic processes. The classical Fourier, the minimum variance and the principal component estimators are some of the suboptimal spectral estimators, which overcome the computational problems of the maximum likelihood estimator. These, however, are limited in resolution by the uncertainty principle arising out of the finite observation interval, as well as by the signal to noise ratios. The more recent eigensubspace methods, notably MUSIC and ESPRIT, have proved to be the most powerful and popular tools for the class

of sinusoidal parameter estimation problems. They exploit the rich subspace structure of the correlation matrices of sinusoids in white noise and produce very high resolution frequency estimates compared to all the other suboptimal methods. Yet they do not suffer from computational difficulties that arise in implementing the ML estimator. Furthermore, from a theoretical viewpoint, the eigensubspace methods are more natural approaches for modeling and spectral parameter estimation of harmonic processes, as compared to the time series based methods. In fact, since harmonic processes are generated by self excited AR systems, these processes cannot be realized as outputs of white noise driven rational models (which is the basic philosophy behind parametric time series modeling). On the other hand, the eigensubspace techniques for power spectrum estimation exploit apriori knowledge of the structure of the autocorrelation matrix of white noise corrupted sinusoids, and as such, are more natural approaches to sinusoidal parameter estimation.

Motivated by these nice properties of the eigensubspace estimators for power spectrum estimation, we have attempted to develop in this thesis, higher order extensions of these for estimating the polyspectra of harmonic processes. In particular, this thesis proposes *new high resolution eigenspace methods for quadratic coupling and bispectrum estimation, and more generally, for polyspectral parameter estimation in noise corrupted harmonic signals.* These techniques exploit the subspace structure of certain "cumulant matrices", just as the eigenstructure methods for power spectral analysis exploit the subspace structure of the correlation matrices. The specific contributions of this thesis are briefly outlined below.

A new structured matrix is first proposed, constructed from the complex third order cumulants of a noisy harmonic process. A *higher order MUSIC algorithm* , based on the complete orthonormal eigenstructure of this matrix, is developed for estimating the phase coupled frequency pairs in the signal. For this purpose, a correspondence between the coupled frequency pairs and certain "steering vectors" in the signal subspace of the cumulant matrix is set up via the Kronecker product map. The algorithm involves constructing a search function of two frequency variables by exploiting this correspondence and the signal-noise subspace orthogonality. We term this function as the MUSIC pseudo-bispectrum, as the location of its peaks coincide with those of the true bispectrum (although the strengths of the peaks do not).

Higher order ESPRIT algorithms for *estimating and quantifying* phase coupling are developed next. These involve, in addition to the cumulant matrix constructed for the MUSIC -like method, two other "shifted versions" of it. ESPRIT - like algorithms based on rotational invariance properties of these shifted cumulant matrices are used to extract the coupled pairs and the coupling strengths from their generalized eigenstructure. Some nice, algebraic interpretations for phase coupling in terms of the dimensionality of the intersection between generalized eigenspaces are derived in the process.

The above methods are subsequently generalized to obtain similar eigensubspace algorithms for extracting the polyspectral parameters of an arbitrary random harmonic process. Specifically, exploiting the *apriori* structure of the higher order statistics of sinusoids in noise, higher order MUSIC and ESPRIT-like techniques are developed for extracting the locations and strengths of the polyspectral impulses in the higher dimensional frequency plane. Apart from providing practical high resolution eigensubspace algorithms for estimating polyspectra, these techniques offer insight into the structure of higher order phase coupling and polyspectra from an algebraic point of view by providing subspace based interpretations for these. These approaches to polyspectral analysis may be regarded as appropriate higher order extensions of the well known correlation matrix based MUSIC and ESPRIT algorithms used in the power spectral analysis of harmonics.

Since, in practice, the above algorithms are implemented by replacing the true cumulant values by their estimated ones, we require to perform a statistical analysis of the cumulant estimates of a harmonic random process from finite data. We derive conditions for a harmonic process to have stationary higher order complex cumulants and moments. Conditions are also derived for the higher order ergodicity, i.e., conditions for convergence of cumulant and moment estimates based on sample averages to the true statistical values.

Upper bounds on the peak values of the absolute errors involved in estimating the higher order moments from a finite data set are derived. The asymptotic values of these error bounds are also obtained. In addition, the corresponding upper bounds on the error variances have been computed. All these bounds are expressed as functions of the data length and the signal parameters (frequencies and amplitude statistics). In the process, we obtain a lower bound on the sample size for accurate estimation of the higher order moments of a harmonic signal. These results are then specialized to third order moments, and special attention is focussed on a harmonic process with phase relations of the quadratic coupling type.

Finally, by invoking the structural similarity between the autocorrelation function of a multidimensional sinusoidal random process, and the higher order cumulants of a one dimensional sinusoidal process, the proposed algorithms for polyspectral estimation have been suitably modified and applied to the multidimensional harmonic retrieval problem. This leads us, in particular, to a matrix pencil solution to the 2-D bearing estimation problem using sensor array triads having almost arbitrary geometries.

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