SOME STUDIES ON NONLINEAR STATIC AND DYNAMIC ANALYSIS OF LAMINATED COMPOSITE RECTANGULAR PLATES SUBJECTED TO THERMOMECHANICAL LOADING

BY

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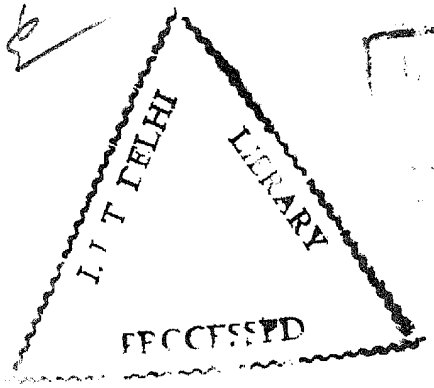
Plates and Shells



Dedicated

to

My Parents

Sri Kapil Deo Shukla & Smt. Shanti Shukla

CERTIFICATE

This is to certify that the thesis entitled " **Some Studies on Nonlinear Static and Dynamic Analysis of Laminated Composite Rectangular Plates Subjected to Thermomechanical Loading"** being submitted by **Mr. Karunesh Kumar Shukla** for the award of the degree of Doctor of Philosophy, to the **Indian Institute of Technology, Delhi,** is a record of bonafide research work carried out by him. The thesis in my opinion has reached the requisite standard fulfilling the requirement of Doctor of Philosophy Degree.

The research report and the results presented in this thesis have not been submitted in parts or in full to any other University or Institute for the award of any degree or diploma.

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(Karunesh Kumar Shukla)

ABSTRACT

With the advent of the fibre production technology, the use of fibre reinforced composite materials in modern engineering structures such as space vehicles, aircraft, submarines, automobiles, and nuclear reactor vessels etc. has increased considerably. This is due to their high directional strength to weight ratio, high stiffness to weight ratio, fatigue life etc. The structural elements of high-speed aircraft, spacecraft etc. are subjected to intense thermal loading due to aerodynamic pressure and solar radiation heating. The future aerospace transportation systems will require structural elements made up of stronger and lighter materials that must withstand very high temperatures. The curved and flat panels are the basic structural elements of these structures. In most of the cases, thermal effects turn out to be more detrimental. Buckling has been one of the predominant considerations in design of composite plate and shell structures. Therefore, a study for estimation of buckling loads and assessment of reserve strength of laminated composite plates/shells under thermal and thermomechanical loading is desirable.

The classical Love-Kirchhoff's hypothesis of plates and shells, which has been extensively used for thin plates and shells, does not adequately describe the behaviour of thick laminated structures. It breaks down in the situations where the ratio of in-plane modulus to transverse shear modulus is high and even in determination of higher modes of vibration. The effect of transverse shear is quite significant for thick and moderately thick plates and the theory that incorporates this effect must be employed for the adequate analysis. The first order shear deformation theory seems to provide the best compromise between accuracy, computational efficiency and efforts.

The governing equations of motion of laminated composite rectangular plates subjected to thermomechanical loading and undergoing moderately large deformations are highly nonlinear and not easily amenable to exact solutions. As it is difficult to obtain analytical solutions, the numerical methods such as finite element, differential quadrature etc. are employed as a necessity. Hence, there is a need for evaluation and improvement of existing methods and development of new approaches to solve these complicated problems. In the present thesis, an attempt is made to solve these problems analytically.

The object of the thesis is to study the nonlinear static and dynamic behaviour and post-buckling response of moderately thick laminated composite rectangular plates subjected to mechanical, thermal and thermomechanical loading, analytically using Chebyshev series technique. The effects of material properties (material orthotropy, coefficient of thermal expansion), plate parameters (aspect ratio, span to thickness ratio), lamination scheme (orientation and number of laminae), and boundary conditions (Levy type and Non-Levy type) on nonlinear static and dynamic behavior and post-buckling response have been studied and the critical loads/temperatures are estimated. The influence of in-plane, normal and rotary inertia on the large amplitude of motion is also studied.

Considering the effects of transverse shear, rotary inertia and employing generalized nonlinear kinematics (Green's tensor), the governing equations of motion of laminated composite rectangular plates are derived and expressed in terms of the displacement functions. A methodology based on double Chebyshev polynomial approximations to analyze the nonlinear boundary and initial value problems in a rectangular domain is developed and presented. The

nonlinear coupled partial differential equations of motion are linearized using quadratic extrapolation technique. The inertia terms are evaluated by employing the Houbolt implicit time marching scheme. Detailed convergence studies have been carried out to establish the accuracy and stability of solution. In order to validate the present methodology of solution the results have been compared with the results available in open literature and reasonably good comparisons are observed. Several new results are presented in the thesis.

Key Words: Analytical solution, Chebyshev polynomials, Nonlinear, Post-buckling, Thermal analysis, Critical temperature, Dynamic analysis, Composite plates, Laminates

The following papers have been published/ communicated from the present thesis

1. K.K.Shukla and Y.Nath, " Nonlinear Analysis of Moderately Thick Laminated Rectangular Plates", **J. Engineering Mechanics, ASCE**., Vol. 126, No.8, pp. 831-838, 2000.
2. K.K.Shukla and Y.Nath, " Analytical solution for buckling and post buckling of angle-ply laminated plates under thermomechanical loading.", **Int. J. Nonlinear Mechanics,** (Accepted and under publication).
3. K.K.Shukla and Y.Nath, "Thermomechanical post buckling of cross-ply laminated rectangular plates.", **J. Engineering Mechanics, ASCE**. (Accepted).

4. Y.Nath and K.K.Shukla, " Post buckling analysis of angle-ply laminated plates under thermal loading.", **Int. J. Mech. Sci.**, (Communicated).
5. Y.Nath and K.K.Shukla, "Post buckling of cross-ply laminated rectangular plates under in-plane thermal loading.", **Int. J. Nonlinear Modelling in Sc. Engrg.** (Communicated).
6. Y.Nath and K.K.Shukla, "Nonlinear transient analysis of moderately thick laminated composite plates.", **J. Sound and Vibrations** (Communicated).
7. K.K.Shukla and Y.Nath, " Nonlinear analysis of moderately thick rectangular plates: An analytical approach.", **Proc. SEC-2000, IIT Bombay**, pp. 359-366.
8. K.K.Shukla and Y.Nath, " Nonlinear static response of laminated composite rectangular plates subjected to thermomechanical loading.", **ICRAMS-2000, IIT Kharagpur**, (Accepted).

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