Parallel Numerical Algorithms for Symmetric Positive Definite Linear Systems

By

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To

My Parents

Certificate

This is to certify that the thesis entitled **Parallel Numerical Algorithms for Symmetric Positive Definite Linear Systems** submitted by **Ms. Sarita** for the award of the Degree of Doctor of Philosophy of the Indian Institute of Technology Delhi, is a record of the original bonafide research work carried out by her under my supervision. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree. The results embodied in this thesis have not been submitted in part or full to any other university or institute for the award of any Degree or Diploma.

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Abstract

We give a matrix factorization for the solution of the linear system $Ax = f$, when coefficient matrix A is a dense symmetric positive definite matrix. We call this factorization as "WW^T factorization". The algorithm for this factorization is given. Existence and backward error analysis of the method are given. The WDW^T factorization is also presented.

When the coefficient matrix is a symmetric tridiagonal matrix, a small modification to the WW^T factorization is given and we call this factorization as "WZ factorization". In this factorization the inner $(n-2) \times (n-2)$ submatrices of W and Z are same as the inner $(n-2) \times (n-2)$ submatrices of W and W^T respectively corresponding to the case when coefficient matrix is symmetric tridiagonal matrix. When combined with partitioning scheme, it renders a divide and conquer algorithm for the symmetric tridiagonal linear systems. We proved the existence of the factorization in the important cases which occur frequently in scientific computations; when the coefficient matrix A is (i) Symmetric positive definite matrix (ii) Symmetric diagonally dominant in addition to the nonsingularity. Solution procedure crucially hinges on the solution of the "reduced system". We proved that reduced system retains the original properties of the original matrix like symmetric positive definiteness, diagonally dominance. The backward error analysis is given. The WDZ algorithm is also designed for symmetric tridiagonal linear systems.

The divide and conquer approach considered above is extended to symmetric banded linear systems with semibandwidth β. In this WZ factorization, the inner $(n - 2\beta) \times (n - 2\beta)$ submatrices of W and Z are same as that of W and W^T respectively corresponding to the case when coefficient matrix is a symmetric band matrix with semibandwidth β. Existence theorems and backward error analysis are given.

A parallel algorithm is presented for the solution of symmetric positive definite block tridiagonal linear systems. The idea in solving symmetric tridiagonal linear systems is extended to symmetric block tridiagonal linear systems. The existence theorems and backward error analysis are given.

A parallel algorithm based on multifrontal technique for WW^T factorization during the solution of large sparse symmetric positive definite linear systems is given. The method is formulated in terms of frontal matrices, update matrices and elimination tree. The method reorganizes the overall factorization of a sparse matrix into a sequence of partial factorization of dense smaller matrices.

All the designed parallel algorithms are implemented on the parallel machine **Sunfire 6800** having 16 processors with Solaris 8.0 operating system. MPI is used as internode communication.

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