

INVESTIGATIONS ON STOKES SINGULARITIES

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INVESTIGATIONS ON STOKES SINGULARITIES

by

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Department of Physics

Submitted

in fulfilment of the requirements of the degree of Doctor of Philosophy

to the



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Dedicated to my parents

Certificate

*This is to certify that the thesis entitled, “**Investigations on Stokes Singularities**”, being submitted by **Gauri Arora**, to the Indian Institute of Technology Delhi, India, for the award of degree of **Doctor of Philosophy** in the Department of Physics is a record of bonafide research work carried by her under my supervision and guidance. She has fulfilled the requirements for the submission of the thesis, which to the best of my knowledge has reached the requisite standards.*

The material contained in the thesis has not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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Abstract

This thesis is a study of polarization structured optical beams that contain various Stokes singularities. Singularities are ubiquitous in nature and can be found in many different forms in nature. Optical singularities are points of dislocations where some parameter that defines the light field is indeterminate. Phase and polarization singularities are examples of optical singularities which are mainly dealt in this thesis. Due to the azimuthal component of the Poynting vector in phase singular beams, these beams are known to carry orbital angular momentum. Polarization singularities are subset of general Stokes singularities. Stokes singular beams are superposition of orthogonally polarized beams wherein at least one of the superposing beams carries a phase singularity. Depending on the topological charge of vortex beams and polarization states of the superposing beams, generation and detection of various Stokes singularities are discussed.

Stokes singular beams are inhomogeneously polarized, and can be represented as a spatial region on the Poincaré sphere. The finite size of the beam limits its occupancy on the Poincaré sphere. A beam that encompasses all possible polarization states in its cross-section is called a full Poincaré beam. There is an increasing interest in generating full Poincaré beams due to their wide applications in various fields of optics. We show that by modulating the weightage of superposing modes in the formation of a particular Stokes singularity, beams with all possible polarization states in its cross-section can be realized. Two new geometric constructions are introduced named linear hybrid order Poincaré spheres to represent such beams as points on the sphere. Various degeneracies associated with polarization singular beams are identified and discussed. It is shown that diffraction in combination with polarization transformation can be used to lift any degeneracy associated with polarization singular beams.

In this thesis, the impact of perturbation on vector field polarization singularities is also investigated. It is shown that the strength of perturbation can be measured by examining the perturbed beam. The probable uses of this technique in studying the chiroptical effects are discussed. Additionally, the thesis also reports novel technique to produce spatially varying lattices of Stokes singularities by tailoring multiple beam interference of vector beams. The exotic polarization structures examined here may offer great advantages in optical trapping, image processing, optical manipulation, chirality measurement, optical communication, and focal beam shaping.

सार

यह थीसिस ध्रुवीकरण संरचित ऑप्टिकल बीम का अध्ययन है जिसमें विभिन्न स्टोक्स विलक्षणताएं होती हैं। व्यक्तित्व प्रकृति में सर्वव्यापी हैं और प्रकृति में कई अलग-अलग रूपों में पाए जा सकते हैं। ऑप्टिकल सिंगुलैरिटीज अव्यवस्था के बिंदु हैं जहां प्रकाश क्षेत्र को परिभाषित करने वाले कुछ पैरामीटर अनिश्चित हैं। चरण और ध्रुवीकरण विलक्षणता ऑप्टिकल विलक्षणता के उदाहरण हैं जो मुख्य रूप से इस थीसिस में निपटाए गए हैं। चरण एकवचन बीम में पोरॉटिंग वेक्टर के अज़ीमुथल घटक के कारण, इन बीमों को कक्षीय कोणीय गति को ले जाने के लिए जाना जाता है। ध्रुवीकरण विलक्षणताएं सामान्य स्टोक्स विलक्षणताओं के उपसमुच्चय हैं। स्टोक्स सिंगुलर बीम ऑर्थोगोनली पोलराइज्ड बीम का सुपरपोजिशन है, जिसमें कम से कम एक सुपरपोजिंग बीम में फेज सिंगुलैरिटी होती है। भंवर बीम के टोपोलॉजिकल चार्ज और सुपरपोजिंग बीम के ध्रुवीकरण राज्यों के आधार पर, विभिन्न स्टोक्स विलक्षणताओं की पीढ़ी और पहचान पर चर्चा की जाती है।

स्टोक्स एकवचन बीम अमानवीय रूप से ध्रुवीकृत होते हैं, जिन्हें पॉइंकेयर क्षेत्र पर एक स्थानिक क्षेत्र के रूप में दर्शाया जा सकता है। बीम का परिमित आकार Poincare क्षेत्र पर इसके अधिभोग को सीमित करता है। एक बीम जो अपने क्रॉस-सेक्शन में सभी संभावित ध्रुवीकरण राज्यों को शामिल करता है, एक पूर्ण पॉइंकेयर बीम के रूप में जाना जाता है। प्रकाशिकी के विभिन्न क्षेत्रों में उनके व्यापक अनुप्रयोगों के कारण पूर्ण पॉइंकेयर बीम उत्पन्न करने में रुचि बढ़ रही है। हम दिखाते हैं कि एक विशेष स्टोक्स विलक्षणता के निर्माण में सुपरपोजिंग मोड्स के वेटेज को संशोधित करके, उनके क्रॉस-सेक्शन में सभी संभावित ध्रुवीकरण राज्यों के साथ बीम को महसूस किया जा सकता है। गोले पर बिंदु के रूप में इस तरह के बीम का प्रतिनिधित्व करने के लिए दो नए ज्यामितीय निर्माणों को लीनियर हाइब्रिड ऑर्डर पॉइंकेयर क्षेत्रों के नाम से पेश किया गया है। ध्रुवीकरण एकवचन बीम से जुड़े विभिन्न अधःपतन की पहचान की जाती है और चर्चा की जाती है। यह प्रदर्शित किया गया है कि ध्रुवीकरण परिवर्तन के साथ संयोजन में विवर्तन का उपयोग ध्रुवीकरण एकवचन बीम से जुड़े किसी भी अधःपतन को उठाने के लिए किया जा सकता है।

इस थीसिस में, सदिश क्षेत्र ध्रुवीकरण विलक्षणताओं पर गड़बड़ी के प्रभाव की भी जांच की जाती है। यह दिखाया गया है कि क्षुब्ध बीम की जांच करके गड़बड़ी की ताकत को मापा जा सकता है। काइरोप्टिकल प्रभावों के अध्ययन में इस तकनीक के संभावित उपयोगों पर भी चर्चा की गई है। इसके अतिरिक्त, थीसिस वेक्टर बीम के कई बीम हस्तक्षेप को सिलाई करके स्टोक्स विलक्षणताओं के स्थानिक रूप से अलग-अलग जाली बनाने के लिए उपन्यास तकनीक की भी रिपोर्ट करती है। इस थीसिस में जांच की गई विदेशी ध्रुवीकरण संरचनाएं और उनके अद्वितीय गुण ऑप्टिकल ट्रैपिंग, इमेज प्रोसेसिंग, ऑप्टिकल हेरफेर, चिरायता माप, ऑप्टिकल संचार और फोकल बीम को आकार देने में बहुत लाभ प्रदान कर सकते हैं।

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