

LIE THEORY AND GENERATING FUNCTIONS

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for the award of the Degree of
Doctor of Philosophy**


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CERTIFICATE

This is to certify that the thesis entitled "Lie Theory and Generating Functions" which is being submitted by Miss Sunita Jain for the award of the degree of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Delhi, is a record of bonafide research work that she has been doing for the last two and a half years under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other university or institute for the award of any degree or diploma.


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PREFACE

In view of the growing importance of generating functions, the piece of work in this thesis contains several generating functions based on different techniques.

It consists of an introductory chapter and four main chapters. A brief summary of the problem is presented at the beginning of each chapter.

The aim of the introductory chapter is to provide a survey of special functions used in the present work.

In chapter II an attempt is made to bring into use the ideas of Lie theory for obtaining generating functions for modified Laguerre polynomial $L_{m,n}^{(\gamma)}(x)$, hypergeometric functions ${}_2F_1[-m-n, \beta, \gamma; x]$, modified Jacobi polynomial $p^{(\alpha-n, \beta-n)}(x)$ and modified Laguerre polynomial $f_{m,n}^{\beta}(x) = (-1)^{m+n} L_{m+n}^{(-\beta-m-n)}(x)$.

In chapter III a new and powerful method based on the orthogonality of the exponentials for obtaining linear, bilinear and bilateral generating functions is introduced.

Then in chapter IV using Cauchy's Integral formula and series manipulation techniques certain results for Jacobi polynomials are obtained.

Finally, in chapter V theorems for generating functions have been given. These theorems being of a general nature unify generating functions of various polynomials and special functions

CONTENTS

CHAPTER		PAGE
	PREFACE	
I	INTRODUCTION	1 - 21
	1.1 Hypergeometric Functions	1
	1.2 Systems of Partial Differential Equations	7
	1.3 Functions of Two and More Variables	8
	1.4 Orthogonal Polynomials	15
	1.5 Generating Functions	17
	References	21
II	GENERATING FUNCTIONS BASED ON LIE GROUPS	22 - 89
	Introduction	
	2.1 Lie Groups: Definitions and Theorems	22
	2.2 Some Lie Groups and Their Lie Algebras	27
	2.3 Homomorphisms	30
	2.4 Linear Differential Operators	32
	2.5 Observations	33
	2.6 Modified Laguerre Functions $L_{m,n}^{(\gamma)}(x)$	35
	2.7 Hypergeometric Functions ${}_2F_1[-n-n, \beta; \gamma; x]$	37
	2.8 Modified Jacobi Functions $P_n^{(\alpha-n, \beta-n)}(x)$	69
	2.9 Modified Laguerre Functions $P_{m,n}^{\beta}$	81
	References	89

III	GENERATING FUNCTIONS BASED ON ORTHOGONAL PROPERTY OF EXPONEN- TIAL FUNCTIONS	90 - 159
	3.1 Theorem for Obtaining the Genera- ting Relations for Product Sequence	90
	3.2 Linear Generating Functions	93
	3.3 Bilinear Generating Functions	120
	3.4 Bilateral Generating Functions	126
	References	157
IV	GENERATING FUNCTIONS INVOLVING JACOBI POLYNOMIALS	140 - 160
	4.1 Single Series Generating Functions	140
	4.2 Double Series Generating Functions	152
	References	160
V	GENERAL THEOREMS ON GENERATING Functions	161 - 190
	Theorem 5.1 on Linear Generating Functions	161
	Theorem 5.2 on Linear Generating Functions	168
	Theorem 5.3 on Linear Generating Functions	173
	Theorem 5.4 on Linear Generating Functions	176
	Theorem 5.5 on Generating Functions for Product of Two Functions	179
	Theorem 5.6 Expansion Formulae	183
	References	190