

ON MODEL PREDICTIVE CONTROL OF SWITCHED LINEAR AND SWITCHED AFFINE SYSTEMS

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ON MODEL PREDICTIVE CONTROL OF SWITCHED LINEAR AND SWITCHED AFFINE SYSTEMS

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CERTIFICATE

This is to certify that the thesis entitled **On Model Predictive Control of Switched Linear and Switched Affine Systems** submitted by **Midhun T. Augustine** to the Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy**, is a record of the bona fide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other University or Institute for the award of any degree or diploma to the best of my knowledge.



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Midhun T. Augustine

ABSTRACT

This thesis deals with the design and analysis of *stabilizing model predictive control (MPC) schemes for switched systems*. We consider two classes of switched systems which are *switched linear systems (SLSs) and switched affine systems (SASs)*. SLSs consists of a finite set of linear time-invariant (LTI) subsystems and a switching index that decides switching among the subsystems. Optimal control of SLSs deals with the problem of computing an optimal switching index and control input schemes, which are to be optimized together for minimizing the cost function. This leads to a mixed-integer optimization problem for which the computational complexity increases exponentially with the number of time instants or switching instants. The initial works in this area study the linear quadratic regulator (LQR) problem of continuous-time SLSs also known as the switched LQR (SLQR) problem. Later on, LQR problems for discrete-time SLSs are studied which is known by the name discrete-time switched LQR (DSLQR). To reduce the computation and storage required for the DSLQR problem, approaches such as dynamic programming with pruning and relaxed dynamic programming are proposed in the past. Recently, model predictive control (MPC) of SLSs is studied which gives suboptimal solutions to the infinite horizon DSLQR problem by solving a finite horizon DSLQR problem in a receding horizon manner. In the MPC of SLSs, the major challenges are ensuring stability (because of its receding horizon nature) and reducing online computation (in the case of large prediction horizons). The stability of the SLS with MPC is not guaranteed in general and the stability depends on the prediction horizon. This motivates our study in SLSs and the main focus of the thesis is to design stabilizing MPC schemes for a class of SLSs with at least one stabilizable subsystem. Firstly, stabilizing MPC schemes are designed for a class of SLSs using Lyapunov inequality-based pruning which permits switching to only stabilizable subsystems. The MPC scheme is then modified to incorporate switching to nonstabilizable subsystems using a pruning based on Lyapunov-Metzler inequality. The stability and feasibility of the MPC schemes are studied and the proposed MPC schemes ensure global exponential stability of the SLSs without constraints. The MPC schemes are also extended to incorporate constraints on states and inputs of SLSs for which stability results are presented. Further, we consider the design of MPC schemes for SASs which are a subclass of switched systems with an affine term. The affine term makes the exponential or asymptotic stabilization of SASs difficult. Consequently, we focus on the practical stabilization of SASs under the MPC scheme. In the proposed approach practical stability is guaranteed for MPC schemes for SASs with arbitrary prediction horizons using an offline pruning algorithm with cost approximation. In addition, the thesis also presents the application of the proposed MPC schemes to networked controls systems and switching power converters.

Keywords: Switched Linear Systems, Switched Affine Systems, Linear Quadratic Regulator, Model Predictive Control, Exponential Stability, Practical Stability, Lyapunov approach, Networked Control Systems, Switching Power Converters.

सार

यह थीसिस स्विच सिस्टम के लिए मॉडल प्रेडिक्टिव कंट्रोल (MPC) योजनाओं को स्थिर करने के डिजाइन और विश्लेषण से संबंधित है। हम स्विच सिस्टम के दो वर्गों पर विचार करते हैं जो स्विच लीनियर सिस्टम (SLS) और स्विच एफाइन सिस्टम (SAS) हैं। SLS में लीनियर टाइम-इनवैरिएंट (LTI) सबसिस्टम का एक सीमित सेट और एक स्विचिंग इंडेक्स होता है जो सबसिस्टम के बीच स्विचिंग का निर्णय लेता है। SLS का इष्टतम नियंत्रण, इष्टतम स्विचिंग इंडेक्स और नियंत्रण इनपुट योजनाओं की गणना करने की समस्या से संबंधित है, जिन्हें लागत समारोह को कम करने के लिए एक साथ अनुकूलित किया जाना है। यह एक मिश्रित-पूर्णांक अनुकूलन समस्या की ओर जाता है जिसके लिए कम्प्यूटेशनल जटिलता समय की संख्या या स्विचिंग इंडेक्स की संख्या के साथ तेजी से बढ़ जाती है। इस क्षेत्र में प्रारंभिक कार्य निरंतर-समय SLS की रैखिक द्विघात नियामक (LQR) समस्या का अध्ययन करते हैं जिसे स्विच LQR (SLQR) समस्या के रूप में भी जाना जाता है। बाद में, डिस्क्रीट-टाइम SLS के लिए LQR समस्याओं का अध्ययन किया जाता है, जिसे डिस्क्रीट-टाइम स्विच LQR (DSLQR) के नाम से जाना जाता है। DSLQR समस्या के लिए आवश्यक संगणना और भंडारण को कम करने के लिए, अतीत में प्रूनिंग के साथ गतिशील प्रोग्रामिंग जैसे दृष्टिकोण प्रस्तावित किए गए हैं। हाल ही में, SLS के मॉडल प्रेडिक्टिव कंट्रोल (MPC) का अध्ययन किया गया है, जो एक परिमित क्षितिज DSLQR समस्या को घटते क्षितिज तरीके से हल करके अनंत क्षितिज DSLQR समस्या का उप-इष्टतम समाधान देता है। SLS के MPC में, प्रमुख चुनौतियां स्थिरता सुनिश्चित कर रही हैं (इसकी घटती क्षितिज प्रकृति के कारण) और ऑनलाइन संगणना को कम करना (बड़े भविष्यवाणी क्षितिज के मामले में)। MPC के साथ SLS की स्थिरता की सामान्य रूप से गारंटी नहीं है और स्थिरता पूर्वानुमान क्षितिज पर निर्भर करती है। यह SLS में हमारे अध्ययन को प्रेरित करता है और थीसिस का मुख्य फोकस कम से कम एक स्थिर उपप्रणाली के साथ SLS के एक वर्ग के लिए MPC योजनाओं को स्थिर करना है। सबसे पहले, MPC योजनाओं को स्थिर करना लायपुनोव असमानता-आधारित छंटाई का उपयोग करते हुए SLS के एक वर्ग के लिए डिज़ाइन किया गया है जो केवल स्थिर उप-प्रणालियों पर स्विच करने की अनुमति देता है। MPC योजना को तब लायपुनोव-मेट्रिक असमानता के आधार पर छंटाई का उपयोग करके गैर-स्थिर उप-प्रणालियों में स्विचिंग को शामिल करने के लिए संशोधित किया गया है। MPC योजनाओं की स्थिरता और व्यवहार्यता का अध्ययन किया जाता है और प्रस्तावित MPC योजनाएं बिना किसी बाधा के SLS की वैश्विक घातीय स्थिरता सुनिश्चित करती हैं। MPC योजनाओं को राज्यों पर बाधाओं और SLS के इनपुट को शामिल करने के लिए भी विस्तारित किया गया है जिसके लिए स्थिरता परिणाम प्रस्तुत किए गए हैं। इसके अलावा, हम SAS के लिए MPC योजनाओं के डिजाइन पर विचार करते हैं जो स्विच सिस्टम का एक उपवर्ग है। एफ़िन टर्म SAS के घातीय या स्पर्शोन्मुख स्थिरीकरण को कठिन बनाता है। नतीजतन, हम MPC योजना के तहत SAS के व्यावहारिक स्थिरीकरण पर ध्यान केंद्रित करते हैं। प्रस्तावित दृष्टिकोण में लागत सन्निकटन के साथ एक ऑफ़लाइन प्रूनिंग एल्गोरिथ्म का उपयोग करते हुए मनमाने ढंग से भविष्यवाणी क्षितिज के साथ SAS के लिए MPC योजनाओं के लिए व्यावहारिक स्थिरता की गारंटी है। इसके अलावा, थीसिस नेटवर्क कंट्रोल सिस्टम और स्विचिंग पावर कन्वर्टर्स के लिए प्रस्तावित MPC योजनाओं के आवेदन को भी प्रस्तुत करती है।

कीवर्ड: स्विच लीनियर सिस्टम, स्विच एफ़िन सिस्टम, लीनियर क्वाड्रैटिक रेगुलेटर, मॉडल प्रेडिक्टिव कंट्रोल, एक्सपोनेंशियल स्टेबिलिटी, प्रैक्टिकल स्टेबिलिटी, लायपुनोव अप्रोच, नेटवर्क कंट्रोल सिस्टम, स्विचिंग पावर कन्वर्टर्स।

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List of Abbreviations

AC	Alternating Current	7
ADP	Approximate Dynamic Programming	100
ARE	Algebraic Riccati Equation	24
BMI	Bilinear Matrix Inequality	49
CQLF	Common Quadratic Lyapunov Function	17
DC	Direct Current	7
DP	Dynamic Programming	22
DRE	Difference Riccati Equation	23
DSLQR	Discrete-time Switched Linear Quadratic Regulator	2
ELF	Exponential Lyapunov Function	17
EVP	Eigenvalue Problem	49
GES	Global Exponential Stability	17
LQR	Linear Quadratic Regulator	2
LMI	Linear Matrix Inequality	49
LTI	Linear Time-Invariant	4
LTV	Linear Time-Varying	15
MPC	Model Predictive Control	1
MPC L	Model Predictive Control using Lyapunov inequality based pruning	40
MPC LM	Model Predictive Control using Lyapunov-Metzler inequality based pruning	48
MPC P	Practically stabilizing Model Predictive Control	65
NCS	Networked Control System	4
PLF	Practical Lyapunov Function	20
PQLF	Piecewise Quadratic Lyapunov Function	18
RHC	Receding Horizon Control	3
SAS	Switched Affine System	1
SLS	Switched Linear System	1
SMC	Sliding Mode Control	9
SLQR	Switched Linear Quadratic Regulator	2
SPC	Switching Power Converter	7
UpAS	Uniform Practical Asymptotic Stable	19

List of Symbols

s	scalar s
\mathbf{s}	vector \mathbf{s}
\mathbf{S}	matrix \mathbf{S}
\mathcal{S}	set \mathcal{S}
\mathbb{N}	Set of natural numbers
\mathbb{Z}	Set of integers
\mathbb{R}	Set of real numbers
\mathbb{Z}^+	Set of nonnegative integers
\mathbb{R}^+	Set of nonnegative real numbers
\mathbb{R}^n	n -dimensional Euclidean space
$\mathbb{R}^{m \times n}$	space of $m \times n$ real matrices
\exists	there exists
\forall	for all
$\mathbf{P} \succ \mathbf{0}$	Positive definite matrix \mathbf{P}
$\mathbf{P} \succeq \mathbf{0}$	Positive semidefinite matrix \mathbf{P}
$\mathbf{P} \prec \mathbf{0}$	Negative definite matrix \mathbf{P}
$\mathbf{P} \preceq \mathbf{0}$	Negative semidefinite matrix \mathbf{P}
$\lambda_{\min}(\mathbf{P})$	minimum eigenvalue of the matrix \mathbf{P}
$\lambda_{\max}(\mathbf{P})$	maximum eigenvalue of the matrix \mathbf{P}
$[\mathbf{s}]_i$	i^{th} element of the vector \mathbf{s}
$\ \mathbf{s}\ $	Euclidean norm of the vector \mathbf{s}
$\ \mathbf{s}\ _p$	p - norm of the vector \mathbf{s}
$[\mathcal{S}]_i$	i^{th} element of the ordered set \mathcal{S}
$ \mathcal{S} $	Cardinality of the set \mathcal{S}
\mathbf{I}_n	Identity matrix with dimension $n \times n$