A THESIS ON

"CERTAIN INVESTIGATIONS IN BRANS-DICKE SCALAR-TENSOR THEORY OF GRAVITATION"

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A C K N O W L E D G E M E N T S

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C E R T I F I C A T E

This is to certify that the thesis entitled "Certain Investigations in Brans-Dicke Scalar-Tensor Theory of Gravitation" that is being submitted by Mr. Dandala Radha Krishna Reddy for the award of the Degree of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Delhi, is a record of bonafide research work carried out by him. He has worked for the last three and half years under my supervision and guidance.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results in this thesis have not been submitted to any other university or institute for the award of any degree or diploma.

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S Y N O P S I S

Since Einstein first published his theory of gravitation a number of modifications have been proposed from time to time which seek to incorporate into the theory certain "desirable" features lacking in the original theory. In recent years there have been some interesting attempts to generalise the general theory of relativity of Einstein incorporating Mach's Principle in a more explicit form than in the general theory. One such attempt which is attracting more and more attention from physicists is the Brans-Dicke Scalar-tensor theory of gravitation which is similar to an earlier attempt by Jordan and his students.

In addition to the metric tensor field Brans-Dicke theory introduces a scalar field which determines the local value of the gravitational constant. The various aspects and consequences of this theory are not fully worked out in view, partly, of the mathematical difficulties of solving complicated field equations.

The investigation of this thesis which is divided into five chapters comprises of certain investigations in Brans-Dicke scalar-tensor theory of gravitation. The purpose of the work done by us is to investigate certain basic consequences of the theory using very elegant mathematical model which was effectively used in case of general relativity first by Weyl

and then by Pauli. The calculations become very much simplified in this model.

In Chapter I, a systematic survey of Brans-Dicke Scalar-tensor theory of gravitation has been conducted. After this, a brief survey of the problems investigated and the relevant literature has been outlined.

Chapter II, mainly deals with the solutions of a vacuum static spherically symmetric metric in Brans-Dicke theory of gravitation. At the outset the basic technique employed in this chapter and in the following chapters is explained. Then, starting from the usual variational principle of Brans-Dicke theory of gravitational field equations for the vacuum static case of spherical symmetry are obtained in Brans-Dicke scalar-tensor theory following the above technique. The field equations being highly non-linear an approximate solution to the second order is presented. Brans and Dicke obtained an exact solution expressing the line element in the isotropic form. But in their case the equations are complicated. Even though ours is an approximate solution, the results to the second order are in agreement with the results of Brans-Dicke and Heckmann.

Chapter III is devoted to the various experimental tests of the Brans-Dicke theory of gravitation. Using the exterior solutions of the field equations of Brans-Dicke theory of gravitation, investigated in Chapter II, we calculate the

gravitational deflection of light, gravitational redshift and rotation of the perihelion of Mercury. It is observed that the results of the above experimental tests are in agreement with the earlier results obtained by Brans-Dicke and Heckmann. Lense-Thirring precession which is the most sensitive test of the Brans-Dicke theory is calculated following Möller. It is seen that the Lense-Thirring precession is exactly the same as in Einstein's theory except for a factor. We see that our results are same as those obtained by Drill.

Chapter IV is concerned with the solutions for the static spherically symmetric metric due to a point charged mass in Brans-Dicke theory of gravitation. Field equations for a static spherically symmetric metric due to a point charged mass are obtained using the usual variational principle in Brans-Dicke theory. The technique employed is the same as that indicated in Chapter II. The field equations in this theory, again, being highly non-linear an approximate solution of the field equations is obtained. This solution is an analog of Reissner-Nordström solution in general relativity. It is interesting to note that an additional term appears in the expression for the electrostatic potential.

In Chapter V a study is made of interior solution in Brans-Dicke theory of gravitation. From the usual variational principle of the field equations for static spherically symmetric homogeneous incompressible fluid model are obtained following

the technique indicated in Chapter II. It is seen that when the Brans-Dicke scalar is constant the field equations give rise to the interior solution of general relativity. The field equations in this case being, again, highly non-linear an approximate solution of the field equations is obtained. This solution can be looked upon as the analog of the interior solution in general relativity in the approximation considered. It is also seen that on the surface of the fluid sphere this solution goes over to the Brans-Dicke exterior solution of Chapter II. Also, when the dimensionless Brans-Dicke coupling constant approaches infinity the solution tends to the general relativity interior solution.

Major part of the above work has resulted in the following papers:

1. M.N. Mahanta and D.R.K. Reddy, "An Approximate Solution of the Vacuum Static Case of Spherical Symmetry in Brans-Dicke Theory" Pub. in J. Math. Phys. (USA), 12,929 (1971).

2. M.N. Mahanta and D.R.K. Reddy, "Approximate Solution for the Static Spherically Symmetric metric due to a Point Charged Mass in Brans-Dicke Theory" Pub. in J. Math. Phys. (USA), 13 708 (1972).

*3. M.N. Mahanta and D.R.K. Reddy, "An Approximate Interior Solution in Brans-Dicke Theory" Commun. to J. Math. Phys. (USA).

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