COUPLED ZIGZAG AND THIRD ORDER MODELS FOR THERMO-ELECTRO-MECHANICAL ANALYSIS OF HYBRID PIEZOELECTRIC BEAMS

by

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Certificate

This is to certify that the thesis entitled "Coupled Zigzag and Third Order Models for Thermo-electromechanical Analysis of Hybrid Piezoelectric Beams" being submitted by Akil Ahmed to the Indian Institute of Technology, Delhi for the award of the degree of Doctor of Philosophy in Applied Mechanics is a record of original bonafide research work carried out by him under our supervision and guidance. The thesis work, in our opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

The results contained in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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Abstract

Two new efficient one-dimensional coupled zigzag theories are presented for piezoelectric hybrid composite and sandwich beams with any lay-up. The axial displacement is approximated as a combination of global third order variation across the thickness with additional layer-wise linear variation. The temperature-rise and the electric potential are approximated sub-layer-wise as piecewise linear across the thickness. The deflection is approximated to account for the transverse normal strain due to the electric field through the transverse piezoelectric coefficient and due to the temperature-rise through the thermal expansion coefficient. The models consider both the axial and transverse electric fields. In model 1, the conditions of zero transverse shear stress at the bottom and the top surfaces and the conditions of its continuity at the layer interfaces are enforced approximately by neglecting the explicit contribution of the electric potential, whereas in model 2 these conditions are enforced exactly. The displacement field is expressed in terms of three primary displacement variables, electric potential variables and the thermal field. The number of the primary displacement variables is independent of the number of layers and identical to those used in the first order shear deformation theory (FSDT) and the third order theory (TOT). The governing coupled equations of motion, charge balance equations and boundary conditions are derived from the extended Hamilton's principle. These theories can accurately model open and closed circuit boundary conditions. The zigzag theory model 2, which incorporates the effect of the transverse thermal expansion coefficient in the approximation of deflection, is a new theory even for the elastic laminated beam for thermal load.

A new smeared beam coupled consistent third order shear deformation theory (CTOT) is developed with cubic approximation for the axial displacement, uniform deflection across the thickness and piecewise linear approximation of the electric potential and the temperature-rise. Unlike the existing third order theory (TOT), the conditions of zero transverse shear stress at the bottom and the top are exactly satisfied including the contribution of the electric potential. The governing equations are obtained from the extended Hamilton's principle.

Analytical Fourier series solutions are developed for the zigzag theory models 1, 2 and the CTOT for simply-supported beams for the static loads, for the natural frequencies and for the steady state

response under harmonic loads with damping. To assess the new models, an exact two-dimensional (2D) piezoelasticity benchmark solution is developed for steady state response of simply-supported beams under harmonic load with and without damping. The results of the new models are compared with the exact 2D piezothermoelasticity solutions and the FSDT solutions. For this purpose a hybrid test beam with highly inhomogeneous lay-up; beams with symmetric cross-ply, antisymmetric cross-ply and angle-ply composite substrate; beams with sandwich substrate and a 2-layer piezoelectric beam are analysed for various thermo-electromechanical loads. The results of zigzag theory model 2 are generally more accurate than those of the FSDT for the global response parameters and for the through-the-thickness variation of displacements, electric field, and stresses. This accurate model would be ideal for the study of active control of smart beams.

For static analysis of hybrid beams under electromechanical loads, a finite element model (FEM) is developed for the zigzag theory model 2 using cubic Hermite interpolation for deflection and electric potential, and linear Lagrange interpolation for the axial displacement and shear rotation. The expressions of the stiffness matrix and the load vector are derived and evaluated in closed form. The FEM program developed is validated by comparison with the analytical solution for the simply-supported beam. The finite element model does not exhibit any shear locking.

The major contribution of this work is the development of coupled zigzag theory model 2 for static and dynamic thermo-electromechanical analysis of beams which considers electric field along axial and transverse directions, and incorporates a novel idea of non-uniform variation of transverse displacement across the thickness taking into account the piezoelectric and thermal strain in the thickness direction. This theory is as efficient as FSDT and TOT and has been shown to be generally very accurate and the finite element model based on it is free of shear locking.

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