

**ADVANCED ROBUST STRATEGIES FOR ATTITUDE  
CONTROL OF SPACECRAFT**

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INDIAN INSTITUTE OF TECHNOLOGY DELHI**

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CONTROL OF SPACECRAFT**

*by*

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**Submitted**

**in fulfilment of the requirements of the degree of Doctor of Philosophy  
to the**



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**OCTOBER 2021**

*This work is dedicated to my parents ...*

*Ammi (Mrs. Umaira)*

*Abbi (Prof. M. S. Jamil Asghar)*

*and to the loving memory of my supervisor...*

*Late Dr. Mashuq-un-Nabi*



# CERTIFICATE

This is to certify that the thesis entitled “**Advanced Robust Strategies for Attitude Control of Spacecraft**”, submitted by **Syed Muhammad Amrr** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** in Electrical Engineering, is a record of the bonafide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the award of the degree.

The results contained in this thesis have not been submitted either in part or in full to any other University or Institute for the award of any degree or diploma to the best of my knowledge.

**Dr. S. Janardhanan (Caretaker Supervisor)**

Department of Electrical Engineering,  
Indian Institute of Technology Delhi.

**Late Dr. Mashuq-un-Nabi (Supervisor)**

Department of Electrical Engineering,  
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# Abstract

Attitude control system (ACS) design is a crucial task for any spacecraft mission. In the last couple of decades, various nonlinear control theories have been developed and extensively employed to design attitude control laws with the primary objective of achieving the desired attitude or time-varying reference trajectory. While designing the attitude control, an ideal ACS must withstand the challenges of model uncertainties, external disturbances (including gravitational force), modal vibrations, actuator faults, input saturation. Moreover, an efficient controller must also provide features like higher accuracy, faster convergence, lesser energy consumption, invariance against uncertainties and faults, feasibility under a constraint network. With the background of above-mentioned challenges, this thesis proposes consolidated attitude control methods aiming to achieve the desired attitude. Meanwhile, the proposed controllers also provide robustness against various uncertainties, alleviation of chattering, minimum use of communication resources, energy efficiency, anti-unwinding design, parameter optimization.

The sliding mode control (SMC) is a well-known robust nonlinear control strategy. However, the primary limitation of a classical SMC is the occurrence of chattering in input, which may propagate into the system and destabilize the system performance. Therefore, in this thesis, the problem of chattering is alleviated by employing three different approaches. The first strategy explores the finite-time disturbance observer (DO) with advanced SMC schemes. The next approach employs the adaptive higher order SMC with parameter optimization. The third methodology uses an alternative to the SMC by exploring the artificial time delay-based

estimation with state feedback control. These three distinct attitude control strategies overcome/reduce the high-frequency component from the input and successfully achieve the desired system response. These proposed techniques render satisfactory performance, and the control laws are updated on a continuous-time framework. This means the control input is executed digitally using a numerical method with a significantly small periodic sampling time. However, these techniques provide unsatisfactory performance if the wireless communication channel (between the controller module and the actuator) has a limited data transmission rate. Consequently, alternative aperiodic control update schemes are needed to reduce the rate of control input updates through the wireless network, which minimizes the computational burden on the onboard processors.

This thesis investigates two concepts under a constraint network, i.e., quantization technique and event trigger (ET) approach, along with the invariant control methods for attitude regulation of spacecraft. In this regard, an adaptive feedback control with an input hysteresis quantizer is employed to achieve a satisfactory response with a minimum usage of control updates. Besides, the proposed ET scheme is investigated with different robust strategies (i.e., high gain and adaptive SMC) under constrained wireless networks. Moreover, a finite time DO with a dynamic ET technique is also proposed to tackle the input chattering and constraint network problems.

The proposed strategies are theoretically investigated by the Lyapunov stability theory and the state trajectories convergence analysis. Furthermore, the efficacy of the proposed schemes is assessed and compared with state-of-the-art through numerical analysis.

**Keywords:** Spacecraft attitude control, sliding mode control, Lyapunov theory, finite-time convergence, disturbance observer, metaheuristic technique, artificial time delay estimation, constraint network, event-trigger technique, hysteresis quantizer.

## सार

एटिट्यूड कंट्रोल सिस्टम (एसीएस) डिजाइन किसी भी अंतरिक्ष यान मिशन के लिए एक महत्वपूर्ण कार्य है। पिछले कुछ दशकों में, वांछित दृष्टिकोण या समय-भिन्न संदर्भ प्रक्षेपवक्र को प्राप्त करने के प्राथमिक उद्देश्य के साथ रवैया नियंत्रण कानूनों को डिजाइन करने के लिए विभिन्न गैर-रेखीय नियंत्रण सिद्धांतों को विकसित और व्यापक रूप से नियोजित किया गया है। एटिट्यूड कंट्रोल को डिजाइन करते समय, एक आदर्श एसीएस को मॉडल अनिश्चितताओं, बाहरी डिस्टर्बेंस (गुरुत्वाकर्षण बल सहित), मोडल कंपनी, एक्चुएटर दोष, इनपुट संतृप्ति की चुनौतियों का सामना करना चाहिए। इसके अलावा, एक कुशल नियंत्रक को उच्च सटीकता, तेज अभिसरण, कम ऊर्जा खपत, अनिश्चितताओं और दोषों के खिलाफ अपरिवर्तनीयता, एक बाधा नेटवर्क के तहत व्यवहार्यता जैसी सुविधाएँ भी प्रदान करनी चाहिए। उपर्युक्त चुनौतियों की पृष्ठभूमि के साथ, यह थीसिस वांछित दृष्टिकोण को प्राप्त करने के उद्देश्य से समेकित रवैया नियंत्रण विधियों का प्रस्ताव करती है। इस बीच, प्रस्तावित नियंत्रक विभिन्न अनिश्चितताओं के खिलाफ मजबूती प्रदान करते हैं, बकबक का उन्मूलन, संचार संसाधनों का न्यूनतम उपयोग, ऊर्जा दक्षता, एंटी-अनइंडिंग डिजाइन, पैरामीटर अनुकूलन, आदि।

स्लाइडिंग मोड कंट्रोल (एसएमसी) एक प्रसिद्ध मजबूत गैर-रेखीय नियंत्रण रणनीति है। हालांकि, शास्त्रीय एसएमसी की प्राथमिक सीमा इनपुट में बकबक की घटना है, जो सिस्टम में फैल सकती है और सिस्टम के प्रदर्शन को अस्थिर कर सकती है। इसलिए इस थीसिस में तीन अलग-अलग तरीकों को अपनाकर बकबक की समस्या को कम किया गया है। पहली रणनीति उन्नत एसएमसी योजनाओं के साथ परिमित समय अशांति पर्यवेक्षक (डीओ) की पड़ताल करती है। अगला दृष्टिकोण पैरामीटर अनुकूलन के साथ अनुकूली उच्च क्रम एसएमसी को नियोजित करता है। तीसरी पद्धति राज्य प्रतिक्रिया नियंत्रण के साथ कृत्रिम समय विलंब-आधारित अनुमान की खोज करके एसएमसी के विकल्प का उपयोग करती है। ये तीन अलग-अलग रवैया नियंत्रण रणनीतियाँ इनपुट से उच्च-आवृत्ति घटक को दूर / कम करती हैं और वांछित सिस्टम प्रतिक्रिया को सफलतापूर्वक प्राप्त करती हैं। ये प्रस्तावित तकनीक संतोषजनक प्रदर्शन प्रदान करती हैं, और नियंत्रण कानूनों को निरंतर समय के ढांचे पर अद्यतन किया जाता है। इसका मतलब है कि नियंत्रण इनपुट को एक संख्यात्मक विधि का उपयोग करके डिजिटल रूप से निष्पादित किया जाता है, जिसमें काफी कम आवधिक नमूना समय होता है। हालाँकि, ये तकनीकें असंतोषजनक प्रदर्शन प्रदान करती हैं यदि वायरलेस संचार चैनल (नियंत्रक मॉड्यूल और एक्चुएटर के बीच) में सीमित डेटा संचरण दर है। नतीजतन, वायरलेस नेटवर्क के माध्यम से नियंत्रण इनपुट अपडेट की दर को कम करने के लिए वैकल्पिक एपेरियोडिक नियंत्रण अद्यतन योजनाओं की आवश्यकता होती है, जो ऑनबोर्ड प्रोसेसर पर कम्प्यूटेशनल बोझ को कम करता है।

यह थीसिस एक बाधा नेटवर्क के तहत दो अवधारणाओं की जांच करती है, अर्थात्, परिमाणीकरण तकनीक और इवेंट ट्रिगर (ईटी) दृष्टिकोण, साथ ही अंतरिक्ष यान के दृष्टिकोण विनियमन के लिए अपरिवर्तनीय नियंत्रण विधियों के साथ। इस संबंध में, नियंत्रण अद्यतनों के न्यूनतम उपयोग के साथ एक संतोषजनक प्रतिक्रिया प्राप्त करने के लिए एक इनपुट हिस्टैरिसिस क्वांटिज़र के साथ एक अनुकूली प्रतिक्रिया नियंत्रण कार्यरत है। इसके अलावा, प्रस्तावित ईटी योजना की विवश वायरलेस नेटवर्क के तहत विभिन्न मजबूत रणनीतियों (यानी, उच्च लाभ और अनुकूली एसएमसी) के साथ जांच की जाती है। इसके अलावा, इनपुट चैटिंग और बाधा नेटवर्क समस्याओं से निपटने के लिए एक गतिशील ईटी तकनीक के साथ एक सीमित समय डीओ भी प्रस्तावित है।

प्रस्तावित रणनीतियों की सैद्धांतिक रूप से ल्यपुनोव स्थिरता सिद्धांत और राज्य प्रक्षेपवक्र अभिसरण विश्लेषण द्वारा जांच की जाती है। इसके अलावा, प्रस्तावित योजनाओं की प्रभावशीलता का आकलन किया जाता है और संख्यात्मक विश्लेषण के माध्यम से अत्याधुनिक के साथ तुलना की जाती है।

**कीवर्ड:** स्पेसक्राफ्ट एटिट्यूड कंट्रोल, स्लाइडिंग मोड कंट्रोल, ल्यपुनोव सिद्धांत, परिमित-समय अभिसरण, डिस्टर्बेंस पर्यवेक्षक, मेटाहेरिस्टिक तकनीक, कृत्रिम समय विलंब अनुमान, बाधा नेटवर्क, इवेंट-ट्रिगर तकनीक, हिस्टैरिसिस क्वांटिज़र।

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# Abbreviations

Abbreviation/Symbol	Description
AAPI	Adding a power integrator
ACS	Attitude control system
ALO	Ant Lion optimization
ET	Event triggered
EI	Energy index
SMC	Sliding mode control
ASMC	Adaptive sliding mode control
ASOSMC	Adaptive second order sliding mode control
SOSMC	Second order sliding mode control
HJI	Hamiltonian Jacobi Isaacs
HOSMC	Higher Order sliding mode control
ISMC	Integral sliding mode control
ISS	Input to state stability
NSFTSMC	Non-singular fast terminal SMC
NSFTSS	Non-singular fast terminal sliding surface
DO	Disturbance observer
NDO	Nonlinear disturbance observer
RNDO	Robust nonlinear disturbance observer

ATDC	Artificial time-delayed control
TDE	Time-delayed estimation
UUB	Uniformly ultimately bounded
PID	Proportional integral derivative
PD	Proportional derivative
PDE	Partial differential equation
ZOH	Zero order hold
$\mathcal{H}_\infty$	H infinity control

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# Nomenclature

Symbol	Description
$\mathbb{R}$	Set of real numbers
$\mathbb{R}^+$	Set of positive real numbers
$\mathbb{R}^j$	Real space of dimension $j$
$\mathbb{R}^{j \times j}$	Real matrix of dimension $(j \times j)$
$(\cdot)^T$	Transpose operation on the argument $(\cdot)$
$\exists$	there exists
$\forall$	for all
$\mathbf{I}_{j \times j}$	Identity matrix of dimension $(j \times j)$
$\mathbf{I} \in \mathbb{R}^{3 \times 3}$	Identity matrix
$\mathbf{P} > \mathbf{0}$ (or $\mathbf{P} < \mathbf{0}$ )	Positive (Negative) definite matrix
$\lambda_{\min}(\mathbf{P})$	Minimum eigenvalue of matrix $\mathbf{P}$
$\lambda_{\max}(\mathbf{P})$	Maximum eigenvalue of matrix $\mathbf{P}$
$\ \mathbf{P}\ $	Euclidean norm of matrix $\mathbf{P}$
$\mathbf{J}_0 \in \mathbb{R}^{3 \times 3}$	Nominal inertia matrix of spacecraft
$\Delta \mathbf{J} \in \mathbb{R}^{3 \times 3}$	Uncertain inertia matrix of spacecraft
$\mathbf{J} \in \mathbb{R}^{3 \times 3}$	Total inertia matrix of spacecraft
$\boldsymbol{\delta} \in \mathbb{R}^{4 \times 3}$	Coupling matrix in flexible spacecraft
$\mathcal{J}_0 \in \mathbb{R}^{3 \times 3}$	$\mathbf{J}_0 - \boldsymbol{\delta}^T \boldsymbol{\delta}$

$\mathbf{q}$ or $\mathbf{e} \in \mathbb{R}^4$	Unit quaternion representation
$\mathbf{q}_v$ or $\mathbf{e}_v \in \mathbb{R}^3$	Vector quaternion representation
$q_0$ or $e_0$	Scalar quaternion representation
$s_{q_0}$ or $s_{e_0}^o$	$\text{sign}(q_0(0))$ or $\text{sign}(e_0(0))$
$q_i$ or $e_i$	$i$ th component of quaternion, for $i = 0$ to $3$
$\mathbf{q}_e \in \mathbb{R}^4$	Relative unit quaternion representation
$\mathbf{q}_{ev} \in \mathbb{R}^3$	Relative vector quaternion
$q_{e0}$	Relative scalar quaternion
$\mathbf{q}_{dv} \in \mathbb{R}^3$	Desired reference vector quaternion
$\mathbf{q}_v^\times \in \mathbb{R}^{3 \times 3}$	skew symmetric matrix of the argument vector $\mathbf{q}_v$
$q_{d0}$	Desired reference scalar quaternion
$\mathbf{w}$ or $\boldsymbol{\omega} \in \mathbb{R}^3$	Angular velocity vector of spacecraft
$\mathbf{w}_e \in \mathbb{R}^3$	Relative angular velocity vector
$\mathbf{w}_d \in \mathbb{R}^3$	Desired angular velocity of reference frame
$\mathbf{s}$ or $\boldsymbol{\sigma} \in \mathbb{R}^3$	Sliding manifold vector
$\mathbf{r} \in \mathbb{R}^3$	White noise vector
$\mathbf{d}_0 \in \mathbb{R}^3$	External disturbance
$\mathbf{d} \in \mathbb{R}^3$	Total disturbance
$\bar{d}$ or $d_{\max} \in \mathbb{R}$	Upper bound of disturbance
$\hat{\mathbf{d}} \in \mathbb{R}^3$	Estimated disturbance
$\tilde{\mathbf{d}} \in \mathbb{R}^3$	Estimated disturbance error
$\mathcal{D} \in \mathbb{R}^3$	Lumped disturbance
$\mathbf{u}$ or $\boldsymbol{\tau} \in \mathbb{R}^3$	Total input torque
$\mathbf{u}_c$ or $\boldsymbol{\tau}_c \in \mathbb{R}^3$	Computed control component
$T(u_i) \in \mathbb{R}$	Quantized control output of $i$ th component

$\mathbf{v} \in \mathbb{R}^3$	Derivative of actual input torque $\boldsymbol{\tau}_c$
$\mathbf{v}_n \in \mathbb{R}^3$	Nominal component of $\mathbf{v}$
$\mathbf{u}_a \in \mathbb{R}^3$	Fault component of control actuator
$\mathbf{E} \in \mathbb{R}^{3 \times 3}$	Actuator effective matrix
$\bar{e}_i \in (0, 1]$	Effectiveness parameter of $i$ th axis actuator
$\mathbf{a} \in \mathbb{R}^3$	Biased additive fault
$U_{\max} \in \mathbb{R}$	Maximum allowable actuation torque
$\mathbf{diag}(\cdot)$	Diagonal matrix of the argument vector ( $\cdot$ )
$\mathbf{sgn}(\cdot)$	Signum function on the argument vector ( $\cdot$ )
$\theta$	Angle of rotation
$\mathbf{R} \in \mathbb{R}^{3 \times 3}$	Rotational matrix
$t_f$	Finite time
$t_k$ or $t_i$	Triggering time instants
$\boldsymbol{\varepsilon} \in \mathbb{R}^3$	Measurement error

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